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Three-Dimensional Vorticity Dynamics in a Swirling Jet

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Abstract

Three-dimensional aspects of vorticity concentration, reorientation, and stretching are investigated in a simplified swirling jet model, consisting of a line vortex along the jet axis surrounded by a jet shear layer with both azimuthal and streamwise vorticity. Inviscid three-dimensional vortex dynamics simulations demonstrate the nonlinear interaction and competition between a centrifugal instability and Kelvin-Helmholtz instabilities feeding on both components of the base flow vorticity. The nonlinear evolution of the flow depends strongly on the initial ratio of the azimuthal and axisymmetric perturbation amplitudes. The long term dynamics of the jet can be dominated by counterrotating vortex rings connected by braid vortices, by like-signed rings and streamwise braid vortices, or by wavy streamwise vortices alone.

1 Introduction

Swirling jets play a crucial role in a variety of applications in such fields as propulsion, combustion, and mixing. At the same time, atmospheric conditions can give rise to swirling flows in nature, with both wake and jet-like axial velocity profiles. Examples concern tornados, dust devils and water spouts. All of the above situations are characterized by a complex interplay of a variety of competing dynamical mechanisms. The axial velocity profiles typically allow for shear induced instabilities similar to those encountered in nonswirling flows. However, the additional presence of swirl can result in an unstable radial stratification, thereby leading to centrifugal instabilities as well. Furthermore, the swirl can give rise to standing or propagating nonlinear inertial waves, similar to the internal waves observed in flows with density stratification. Finally, under certain conditions swirling jets are known to produce vortex breakdown events, an important generic phenomenon for which a universally accepted explanation is still elusive. An improved understanding of these mechanisms and their mutual coupling is a prerequisite for the successful development of active and passive control strategies employing sound, nozzle geometry and motion, or micromachines, with the goal of tailoring the flow such as to generate the desired operating conditions.

An introduction into the basic physics of swirling flows is given in [14], while some more advanced aspects pertaining mostly to confined flows are reviewed in [9]. Early analytical investigations were mostly directed at finding similarity solutions to simplified equations and boundary conditions [39], and at determining the linear stability of various combinations of axial velocity profiles and swirl, e.g., Burggraf and Foster [6], Stewartson [35], Leibovich and Stewartson [17], Toplosky and Akylas [36], and Foster [11]. Experimental investigations of swirling jets for the most part have addressed the issue of mean flow profiles and averaged turbulent transport properties, e.g., Chigier and Chervinsky [8], Farokhi et al. [10], Frey and Gessner [12], Mehta et al. [27]. Only recently

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have researchers begun to pay attention to the dominant role played by the underlying vortical flow structures and their dynamical evolution, e.g., Panda and McLaughlin [32]. These authors point out the crucial role played by axisymmetric and helical instability waves, emphasizing the importance of a *structure-based* understanding of the flow dynamics.

To our knowledge, no axisymmetric or three-dimensional nonlinear computations aimed at elucidating the fundamental dynamics of swirling jets are reported in the literature. However, the recent axisymmetric computational results obtained by Lopez [20], Brown and Lopez [5], and Lopez and Perry [21] for an internal swirling flow, and by Krause and colleagues for the vortex breakdown phenomenon (reviewed by Althaus et al. [1]) suggest that such computations can provide some fundamental insight into the flow physics of swirling jets.

For the purpose of studying the nonlinear dynamical interaction of shear and centrifugal instabilities in swirling jets, we recently introduced a simplified model [24] that is an extension to earlier ones proposed by Batchelor and Gill [3], Rotunno [33], and Caflisch, Li and Shelley [7]. It lends itself well to analytical linear stability calculations, as well as to nonlinear Lagrangian vortex dynamics simulations. The model consists of an axial centerline vortex, which is surrounded by a nominally axisymmetric vortex sheet containing both streamwise and circumferential vorticity. While this model has obvious limitations when it comes to reproducing the detailed features of experimentally generated, and often geometry dependent velocity profiles, its simplicity offers several advantages. First of all, it allows for some analytical progress [24] in terms of a straightforward linear stability analysis, which illuminates the competition of centrifugal and Kelvin-Helmholtz instability waves. In particular, the results show that centrifugally stable flows can become destabilized by sufficiently short Kelvin-Helmholtz waves. Secondly, the model enables us to study the nonlinear interaction and competition of the various instability mechanisms involved, by means of fully nonlinear numerical calculations.

Some preliminary nonlinear simulations for *axisymmetric* perturbations were reported by Martin and Meiburg [25], who showed that, under certain circumstances, *counterrotating* vortex rings emerge in the braid regions between the primary vortex rings generated by the Kelvin-Helmholtz instability of the axisymmetric shear layer. These counterrotating vortex rings can trigger a dramatic decrease in the local jet diameter. A further interesting observation shows the circulation of the swirling vortex rings to be time-dependent, in contrast to the vortex rings found in nonswirling jets. The dynamics of these swirling vortex rings represents an interesting research area in its own right. While nonswirling rings have been the subject of considerable theoretical, experimental, and computational research ([34] and references therein), much less is known about vortex rings with swirl, in part because of the considerable difficulties one encounters when trying to generate them experimentally. On the other hand, several recent theoretical investigations addressing the form and stability of isolated swirling vortex rings (Moffatt [31], Turkington [37], Lifschitz and Hameiri [19], Virk et al. [38]) can be expected to stimulate further efforts in this direction.

After a brief discussion of the flow model in Section 2, we will extend our earlier axisymmetric numerical investigation to fully three-dimensionally evolving swirling jets, by imposing azimuthal perturbations in addition to the axisymmetric ones. The azimuthal perturbations can trigger additional instabilities of the rings or the braid regions. The simulations to be discussed will then allow us to investigate the nonlinear interplay of the competing instabilities for various values of the governing dimensionless parameters.

2 Flow model and numerical technique

The present flow model of an axial line vortex surrounded by a nominally axisymmetric shear layer containing streamwise and circumferential vorticity represents an extension of earlier ones investigated by several researchers, dating back to the analyses by Batchelor and Gill [3] as well as Rotunno [33] of the stability of an axisymmetric layer of circular or helical vortex lines. More recently, Caflisch, Li and Shelley [7] introduced the effect of swirl by placing the additional line vortex at the center of the axisymmetric layer. However, their unperturbed vortex sheet had an axial vorticity component only, so that a jet-like velocity component was absent. In the present investigation, we employ a slightly more complicated model (Fig. 1), consisting of a line vortex of strength Γ_c at radius $r = 0$, surrounded by a cylindrical vortex sheet at $r = R$. The unperturbed axisymmetric vortex sheet contains both azimuthal vorticity (corresponding to a jump ΔU_x in the axial velocity) and streamwise vorticity (representing a jump ΔU_θ in the circumferential velocity). The strength of the vortex sheet is taken to be equal and opposite to that of the line vortex. The vortex lines in the sheet hence are of helical shape, with their pitch angle ψ being

$$\psi = \tan^{-1}\left(\frac{\Delta U_\theta}{\Delta U_x}\right) \tag{1}$$

Notice that, in the unperturbed state, the azimuthal velocity outside the jet shear layer vanishes.

By introducing both axial and azimuthal vorticity along with the central line vortex, this model allows for the investigation of competing Kelvin-Helmholtz and centrifugal instabilities, which can be expected to lead to interesting nonlinear dynamical behavior. For the nonswirling top hat jet velocity profile it is known that axisymmetric and helical perturbations will result in the formation of vortex rings or helices, respectively, all of the same sign [22, 23]. For purely swirling flow, on the other hand, Caflisch et al. [7] demonstrated that axisymmetric perturbations lead to the emergence of counterrotating vortex rings. By superimposing a top hat streamwise velocity profile upon the purely swirling flow, we hence expect a breaking of the symmetry exhibited by the purely swirling flow alone. Conversely, introducing swirl into the nonswirling jet flow should lead to a tendency to form azimuthal vorticity of a sign opposite to that of the vortex rings which evolve as a result of the pure Kelvin-Helmholtz instability. Additional azimuthal disturbances will render the flow field fully three-dimensional. Both the swirling vortex rings as well as the braid regions connecting them may develop instabilities that can lead to the formation of concentrated streamwise vorticity.

In order to compute the nonlinear evolution of the flow in response to certain imposed perturbations, we employ a vortex filament technique that is essentially identical to the one used in earlier investigations of plane shear layers, wakes, and jets (Ashurst and Meiburg [2], Meiburg and Lasheras [29], Lasheras and Meiburg [16], Martin and Meiburg [22, 23, 25]). It is based on the theorems of Kelvin and Helmholtz and follows the general concepts reviewed by Leonard [18] and Meiburg [28]. A detailed account of the technique is provided in these earlier references.

For the numerical simulations of the simple jet model, we limit ourselves to the temporally growing problem, in spite of the spatial growth exhibited by a typical experimental flow field. Previous experience concerning the simulation of nonswirling and swirling jets justifies this approach, as it demonstrates that centrifugal and shear instabilities represent the dominant mechanisms in the evolution of the jet. These mechanisms are captured by the temporally growing flow, so that we can expect to gain significant insight into the dynamical evolution of these flows on the basis of the temporal growth approach. Under this assumption, we can concentrate the numerical resolution on one streamwise wavelength, which allows us to take the calculation farther in time. The wavelength in the axial direction, i.e., the length of the control volume, is based on Michalke and Hermann's stability analysis [30] for the spatially evolving nonswirling jet. By using Gaster's transformation [13], we obtain the wavelength of maximum growth for the temporally evolving flow as approximately 2π.

One streamwise wavelength is typically discretized into 105 filaments. Each filament initially contains 123 segments in the circumferential direction. These numbers emerged from test calculations, in which we refined the discretization until a further increase in resolution resulted in very small changes. The Biot-Savart integration is carried out with second order accuracy both in space and in time by employing the predictor-corrector time-stepping scheme, in conjunction with the trapezoidal rule for the spatial integration. As the flow structure develops nonlinearly, the vortex filaments undergo considerable stretching. To maintain an adequate resolution, the cubic spline representation of the filaments is used to introduce additional nodes, based on a criterion involving distance and curvature [2]. Furthermore, the time-step is repeatedly reduced as local acceleration effects increase. The filament core radius σ decreases as its arclength increases, to conserve its total volume.

We take the streamwise velocity difference between the centerline and infinity as the characteristic velocity. The thickness of the axisymmetric shear layer serves as the characteristic length scale, which results in the filament core radius $\sigma = 0.5$. The nominal jet radius R is taken to be 5, and we obtain the ratio of jet radius R and momentum thickness θ of the jet shear layer as $R/\theta = 22.6$. Hence, the ratio $R/\sigma \gg 1$, and we are well within the range of validity of the filament model.

3 Results

It is common to quantify the effect of swirl in terms of a swirl number S [32]

$$S = \frac{\dot{G}_\theta}{R\dot{G}_x} \quad , \quad \dot{G}_\theta = 2\pi \int_0^\infty \rho U W r^2 dr \quad , \quad \dot{G}_x = 2\pi \int_0^\infty \left(\rho U^2 + p\right) r dr \tag{2}$$

which gives the ratio of the axial flux of tangential momentum $\dot{G}_\theta$ to the product of the radius R and the axial flux of axial momentum $\dot{G}_x$. For a vanishing jet shear layer thickness, the above integration over the unperturbed initial velocity profile can be carried out analytically. However, the result depends on the selected reference frame. When compared to the experimental situation of a swirling jet entering a large body of fluid at rest, the proper computational reference frame is the one in which the jet fluid has unit velocity in the streamwise direction, with the fluid outside the jet being at rest. We then obtain

$$S = \frac{\Gamma_c}{2\pi R} \tag{3}$$

However, it is clear from the above that this definition of the swirl number is not very meaningful in characterizing the effect of swirl in the present flow model, because it does not take into account the presence of streamwise vorticity in the axisymmetric shear layer, which is the cause for the centrifugal instability.

The axisymmetric nature of the calculation described by Martin and Meiburg [25] permits the evolution of concentrated vortical structures only in ring-like form. However, it is well known that three-dimensional perturbations to nominally two-dimensional (Bernal and Roshko [4], Ashurst and Meiburg [2], Lasheras and Choi [15], Meiburg and Lasheras [29]) or axisymmetric (Martin and Meiburg [22, 23]) shear flows give rise to concentrated streamwise vortical structures that are predominantly located in the braid region. In order to investigate possible mechanisms for the generation of such structures in swirling jets, we introduce an azimuthal perturbation in addition to the axisymmetric one described above. Just like the axisymmetric perturbation, the azimuthal disturbance displaces the vortex filament centerline radially from its nominal location. In an experiment, this type of perturbation can be imposed, for example, by means of a corrugated nozzle [26]. Due to the nonlinearity of the overall problem, the initial perturbation amplitude ratio represents an important parameter of the problem, as it can favor the rapid growth of one instability over others in their mutual competition. In particular, the early growth of one instability can affect the base flow in such a way as to suppress or accelerate the development of others. It should be pointed out that for this fully three-dimensional case, a Kelvin-Helmholtz instability of the streamwise vorticity can develop, in addition to the Kelvin-Helmholtz instability of the azimuthal vorticity and the centrifugal instability of the streamwise vorticity.

In the first one of the fully three-dimensional simulations, the ratio of the azimuthal and axial velocity jumps across the jet shear layer has the value $\Delta U_\theta/\Delta U_x = 8.2$. The azimuthal disturbance, which has the form of a radial displacement of the filament centerline, has the relatively small amplitude of $\epsilon_2 = 0.002$, while the axisymmetric perturbation, also in the form of a radial filament displacement, has an amplitude of $\epsilon_1 = 0.25$. The wavenumber of the azimuthal perturbation is taken to be five. As can be seen from Fig. 2, the flow again develops two counterrotating vortex rings, as it did for the purely axisymmetric case described in [25]. However, already the side view at $t = 0.977$ shows a slight nonuniformity in the azimuthal direction. By $t = 1.187$, concentrated streamwise braid structures have begun to form, as a result of a Kelvin-Helmholtz instability of the streamwise braid vorticity. It is interesting to note that these braid vortices form only in the braid section *upstream* of the primary rings, i.e., in the narrow part of the braid. In contrast, the widening half of the braid region downstream of the primary rings does not exhibit any signs of concentrated streamwise vortical structures. The explanation for this behavior can be found in the effective wavelength change of the azimuthal Kelvin-Helmholz instability due to the radial velocity component. In those regions where the braid circumference grows, the growth of the Kelvin-Helmholtz instability in the circumferential direction is slowed down as its effective wavelength increases, whereas in the narrowing braid sections the instability is accelerated due to the wavelength reduction.

It is important to point out that the streamwise vortex structures are all of the same sign, i.e., they are *corotating*. The reason for this lies in the fact that the braid vorticity, which forms the streamwise structures by a process of concentration as a result of a Kelvin-Helmholtz instability, is of the same sign everywhere. In this aspect, the evolution of the braid region resembles the situation encountered in a nonswirling jet disturbed by a helical and an azimuthal wave [23]. In contrast to the counterrotating vortex rings, the circulation of the streamwise braid vortices cannot grow without bounds. Rather, it is limited by the fact that, within a $x = const.$ cross section of the jet, the circulation of the jet shear layer vorticity has to be equal and opposite to that of the centerline vortex. Consequently, the maximum strength of the streamwise braid vortices, achieved if all the jet shear layer vorticity is contained in these concentrated structures, is equal to the circulation of the centerline vortex divided by the azimuthal wavenumber.

The above observations are confirmed by the isosurface plot of the vorticity magnitude in Fig. 3 for $t = 1.343$. It shows the counterrotating vortex rings, connected in one half of the braid region by concentrated streamwise vortical structures. A tendency of the braid vortices to wrap around the vortex rings is visible as well.

Figure 4 shows the evolution of a flow if the azimuthal perturbation amplitude is increased to $\epsilon_2 = 0.05$, while the axisymmetric disturbance amplitude is left unchanged at $\epsilon_1 = 0.25$. This increase in the perturbation amplitude ratio is expected to lead to an increased growth of the azimuthal Kelvin-Helmholtz instability, and consequently to a more rapid evolution of concentrated streamwise vortical structures. In addition, the ratio of the azimuthal and axial velocity jumps across the jet shear layer is reduced to $\Delta U_\theta/\Delta U_x = 3.0$. In this way, the development of the centrifugal instability, and with it the formation of the counterrotating ring, is slowed down. As a result, at time 2.402 we recognize concentrated primary vortex rings, connected by strong streamwise braid vortices that now extend over the *entire* length of the braid region. This early formation of streamwise vortices has preempted any coherent directional reversal of the vortex filament portions, so that counterrotating vortex rings have not formed in this flow. However, the centrifugal instability still causes the braid vortices themselves to acquire a strong azimuthal component, thereby generating a staggered pattern, as can be seen at $t = 3.154$. This is confirmed by the three-dimensional isosurface plot, which shows primary vortex rings connected by strong wavy streamwise braid vortices.

This evolution of the flow for $\epsilon_2 = 0.05$ is a typical result of the above mentioned competition between the various instability mechanisms. Under these conditions, the growth of the Kelvin-Helmholtz instability of the streamwise vorticity is accelerated compared to that of the centrifugal instability, so that nearly all of the braid vorticity between the primary vortex rings becomes concentrated in streamwise vortices before counterrotating rings can form.

Figure 5 shows results for $\Delta U_\theta/\Delta U_x = 3.0$ and $\epsilon_2 = 0.25$, i.e., for an even higher azimuthal perturbation amplitude. As a result, the growth of the Kelvin-Helmholtz instability in the azimuthal direction is further amplified, so that now even the primary vortex rings develop only very weakly. Already at $t = 1.543$, strong and slightly wavy streamwise vortices have formed, and a weak tendency towards the formation of the primary rings is visible. At $t = 3.105$ we recognize that the wavy sections of the streamwise structures align themselves in such a way that, together, they nearly form a ring-like structure at the locations where primary rings should develop, although they remain disconnected. Consequently, the three-dimensional isosurface plot shows that for the present flow parameters the swirling jet is dominated by wavy streamwise structures, while neither primary nor secondary counterrotating vortex rings seem to play an important role.

4 Summary and conclusions

The dynamical evolution of swirling jets is characterized by the complex nonlinear interaction of several different competing instability mechanisms. The axial, jet-like velocity profile gives rise to a Kelvin-Helmholtz instability of the azimuthal vorticity component, thereby favoring the formation of like-signed vortex rings, as is well known from investigations of nonswirling jets. However, the additional azimuthal velocity component of the base flow introduces streamwise vorticity as well, whose existence allows for further instabilities to develop. Firstly, there is the possibility for a centrifugal instability to arise, which tends to promote the evolution of counterrotating vortex rings; i.e., rings of both the same as well as of opposite sign compared to those found in nonswirling jets. Secondly, if the streamwise vorticity is mainly concentrated in a narrow shear layer surrounding the jet axis, it can also be subject to a Kelvin-Helmholtz instability in the *azimuthal* direction, which can lead to the evolution of concentrated streamwise vortices.

In order to gain some insight into the nonlinear mechanisms of interaction and competition between these various potential instabilities, we performed nonlinear, inviscid, three-dimensional vortex dynamics simulations for a simplified model of swirling jets. The nature of the model is such that it allows for the easy identification of the various mechanisms at work. By tracking the nonlinear evolution of vortex lines, it enables us to investigate the effects of the centrifugal instability, as well as of the Kelvin-Helmholtz instabilities feeding on both the azimuthal and the streamwise vorticity, onto processes of concentration, reorientation, and stretching of vorticity. The drawback of the present model is that it does not have easily adjustable parameters allowing for the representation of the wide variety of experimentally generated, and often geometry dependent, base flow profiles. In particular, a study of the dynamics of very smooth, Gaussian-like streamwise and azimuthal velocity profiles will have to be based on the evolution of a more continuous initial vorticity distribution, rather than the shear layer model employed here. With this in mind, the current investigation has to be seen as a first step, intended to provide qualitative information on a variety of dynamical mechanisms and their interactions, and to be followed by three-dimensional Navier-Stokes or vortex particle simulations. Nevertheless, the present model elucidates many of the key features expected to dominate the evolution of realistic swirling jets.

A main goal lies in the investigation of the mechanisms by which the introduction of swirl affects the dynamics observed earlier for nonswirling jets [22, 23]. Conversely, the question arises as to how the purely swirling flow examined by Caflisch, Li, and Shelley [7] is modified by the addition of an axial velocity component. We find that the main effect of the added streamwise velocity lies in the breaking of the symmetry of the pure swirling flow. As a result, the counterrotating rings observed in the purely swirling flow are no longer of equal strength, as one of them is amplified, and the other one weakened, by the Kelvin-Helmholtz instability of the axial flow. On the other hand, the introduction of swirl drastically alters the dynamics of nonswirling jets, as it results in the formation of counterrotating vortex rings, whose circulations, in the absence of viscous effects, can grow in time without bounds. These rings promote a pinch-off mechanism leading to a dramatic decrease in the local jet diameter.

While the above mechanisms can be observed in axisymmetric swirling jets, an additional azimuthal perturbation leads to the formation of concentrated streamwise vortices as a result of a Kelvin-Helmholtz instability feeding on the streamwise jet shear layer vorticity. In contrast to nonswirling jets, the streamwise vortices in swirling jets are all of the same sign. The nature of the large scale vortical structures dominating the long term dynamics of the jet depends strongly on the ratio of the initial perturbation amplitudes in the azimuthal and streamwise directions. If this ratio is small, the centrifugal instability has enough time to form counterrotating vortex rings, before concentrated streamwise vortices can emerge in the braid regions between them. For a somewhat larger perturbation amplitude ratio, streamwise vortices grow more rapidly in the braid region between the like-signed primary vortex rings. In this way, they suppress the growth of the counterrotating rings. However, the centrifugal effects lead to a partial reorientation of the braid vortices in the azimuthal direction. Finally, for even larger initial azimuthal perturbation amplitudes, the streamwise vortices grow fast enough to suppress the growth of even the primary corotating vortex rings. In this case, the long term dynamics of the swirling jet is dominated by wavy like-signed axial vortical structures.

The above description is predominantly qualitative, and a more detailed quantitative investigation of these effects is clearly necessary. In particular, it will be of interest to study the competition between the different instability mechanisms as a function of the detailed shape of the base flow profile. Furthermore, the effect of helical rather than axisymmetric perturbations, and their interaction with azimuthal disturbances, needs to be addressed. It is hoped that an investigation along those lines might also help to shed some light onto the various forms of vortex breakdown observed in swirling jets.

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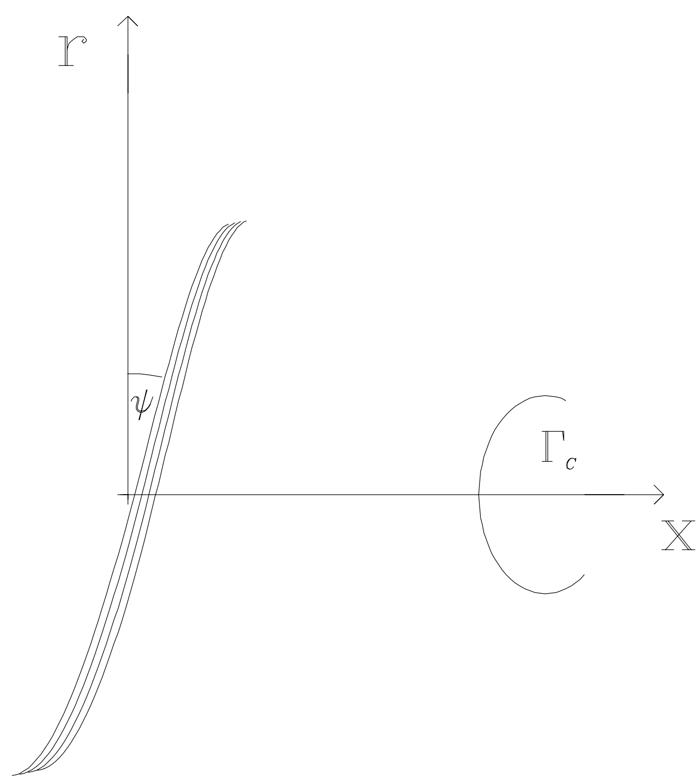

Figure 1: Simplified model of a swirling jet flow. The centerline vortex of strength Γ_c is surrounded by a nominally axisymmetric jet shear layer containing helical vortex lines of pitch ψ. The azimuthal vorticity component is related to the top hat axial velocity profile, whereas the streamwise vorticity component results in the centrifugally unstable stratification.

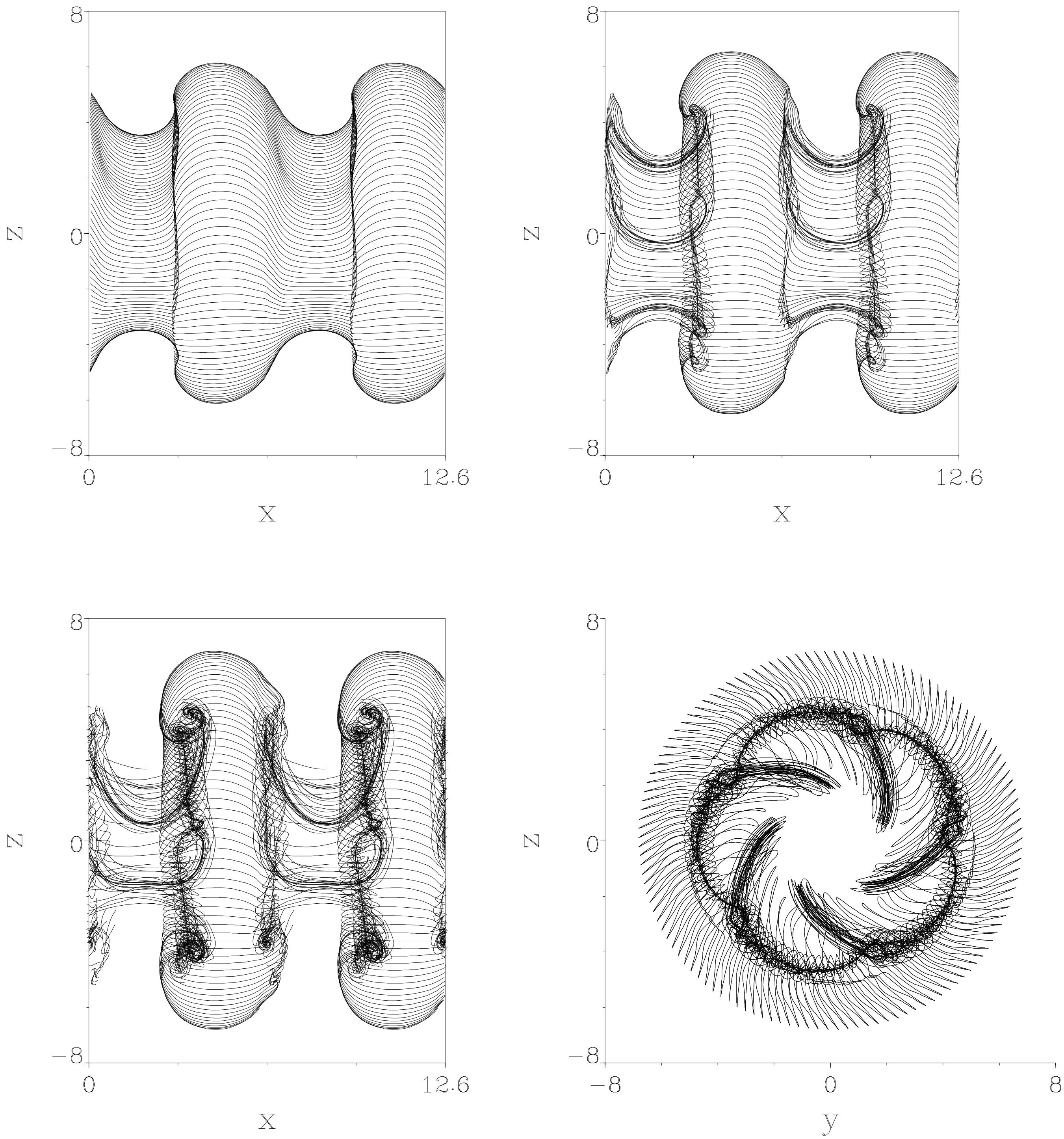

Figure 2: Evolution of a swirling jet with $\Delta U_\theta/\Delta U_x = 8.2$ subject to an axisymmetric perturbation of amplitude 0.25, and an azimuthal disturbance of amplitude 0.002. Shown are side views of the vortex filament centerlines at times 0.977, 1.187, and 1.343, along with an end view for $t = 1.343$. The formation of the primary and counterrotating rings proceeds similarly to the axisymmetric case. However, the azimuthal disturbance leads to the formation of additional concentrated streamwise vortical structures in the narrow half of the braid region.

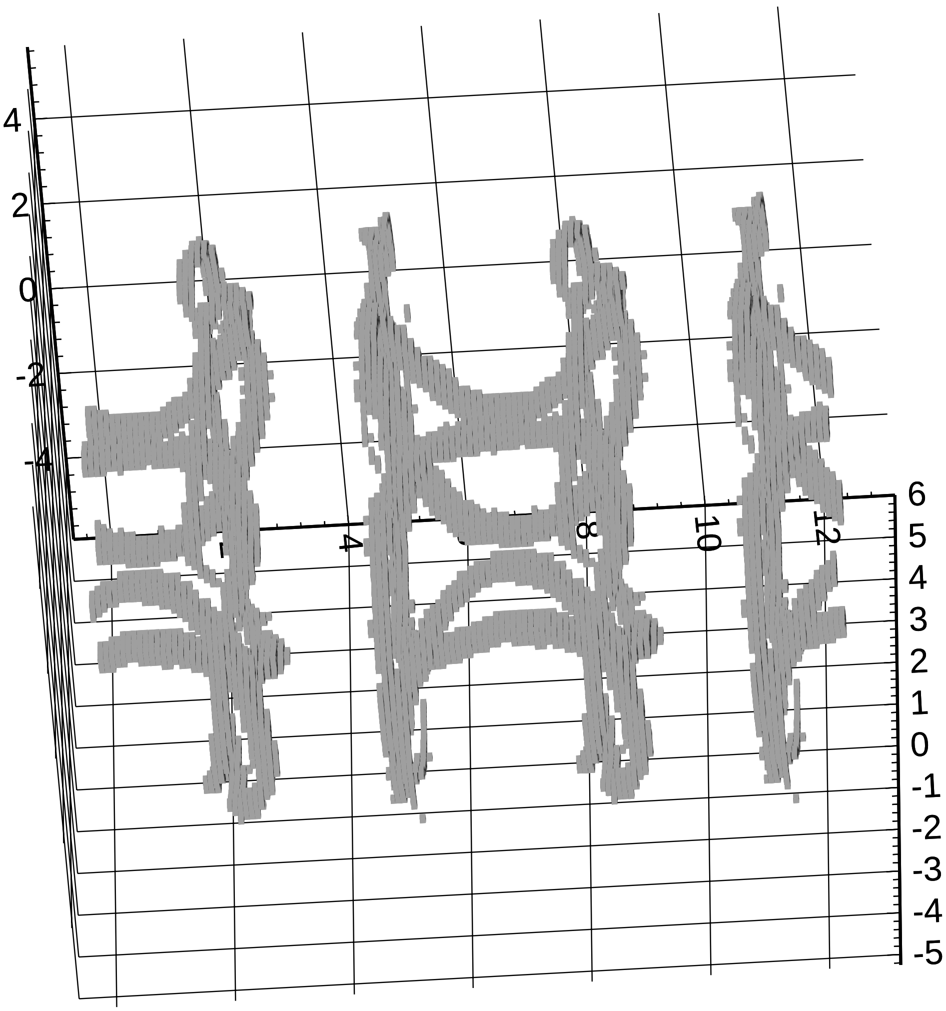

Figure 3: Isosurface plot of the vorticity magnitude for the flow shown in figure 3 at $t = 1.343$. The dominant large scale coherent vortical structures have the form of primary and secondary vortex rings, with additional streamwise vortices located in the narrow section of the braid region.

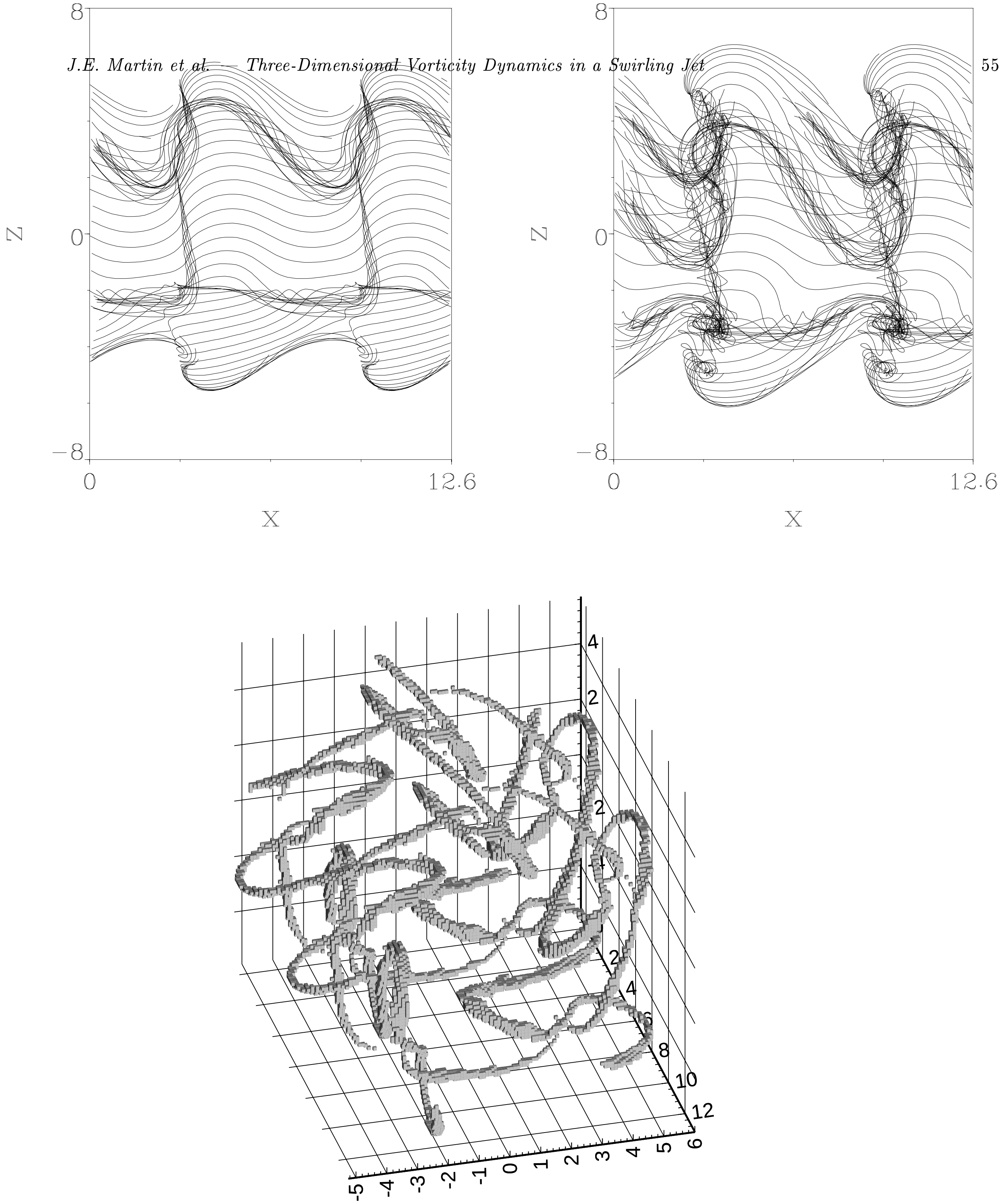

Figure 4: Evolution of a swirling jet with $\Delta U_\theta/\Delta U_x = 3$ subject to an axisymmetric perturbation of amplitude 0.25, and an azimuthal disturbance of amplitude 0.05. Shown are side views at times 2.402 and 3.154, along with a vorticity magnitude isosurface plot for this later time. For these parameters, the streamwise vortical structures develop more rapidly, and they prevent the formation of the secondary counterrotating vortex rings. The isosurface plot shows the dominant large scale structures to have the form of distorted vortex rings, connected by wavy streamwise vortical structures.

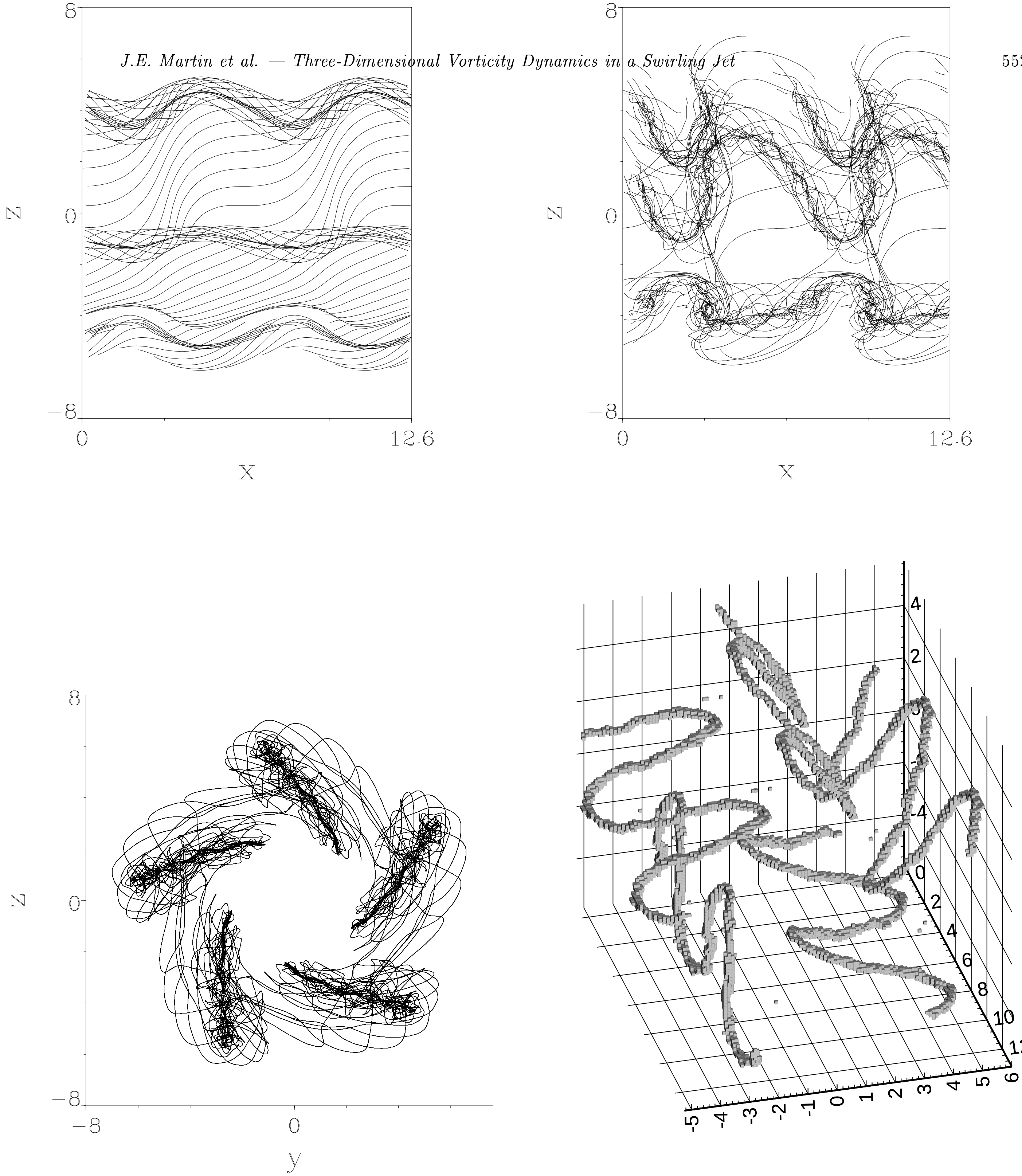

Figure 5: Evolution of a swirling jet with $\Delta U_\theta/\Delta U_x = 3$ subject to an axisymmetric perturbation of amplitude 0.25, and an azimuthal disturbance of amplitude 0.25. Shown are side views at times 1.543 and 3.105, along with an end view and a vorticity magnitude isosurface plot for this later time. Here, the streamwise vortical structures develop even more rapidly, thereby suppressing even the formation of strong primary vortex rings. Consequently, the isosurface plot shows wavy streamwise vortices to dominate the large scale features of the flow.