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# IDENTIFICATION OF THE DYNAMICS OF A LEAD ACID BATTERY BY A DIFFUSIVE MODEL

GERMAIN GARCIA*, JACQUES BERNUSSOU

Abstract. In this paper, some preliminary results on identification of diffusive models using input/output measurements are proposed. The idea consists in considering a finite dimensional approximation of the diffusive model and formulating an optimisation problem of a least square error type. To identify such a model, it is necessary to estimate a distribution $\mu$ which fully characterises the system dynamics. This distribution constitutes the unknown of the problem. These results are applied to the model identification of a lead acid battery.

Résumé. Dans cet article, des résultats préléminaires sur l'identification de modèles diffusifs à partir de mesures entrée/sortie sont proposés. L'idée consiste à prendre une approximation de dimension finie du modèle diffusif et à poser le problème en terme de problème d'optimisation du type "moindres carrés". Plus concrétement, il s'agit d'estimer une distribution $\mu$ qui caractérise complètement la dynamique du système. Cette distribution constitue l'inconnue du problème. Ces résultats sont appliqués au problème d'identification du modèle entrée/sortie d'une batterie PbO2/SO4.

## 1. Introduction

Diffusive representation was initially introduced in [1] with the aim of representing fractional operators in a state space model formulation, the state belonging to an appropriate Hilbert space.

Practically there exist, numerous systems or phenomena with long memory dynamics which can be accurately modelised by fractional integrodifferential operators. In the context of control theory, the representation through a state space model offers an interesting setting both for modelling or controller design problems [1], [2]. Moreover, this description happens in an infinite dimensional framework and convergent approximations are possible and well-suited [2].

Modelling a real system is a difficult task which can be solved in some cases, using identification methods. In the context of finite dimensional linear systems, there exists an important literature, and numerous methods have proven to be efficient [4]. In the one of non integer order linear systems, an extension of classical identification methods was done in [10], [11], [9] defining ARX and ARMAX models to the non integer case.

In this paper, some results on identification of diffusive models using input/output measurements are proposed. The idea consists in considering a finite dimensional approximation of the original state space diffusive model and formulating an optimisation problem of a least square error type. To identify such a model, it is necessary to estimate a distribution $\mu$ which fully characterise the system dynamics. This distribution constitutes the unknown of the problem.



G.Garcia*, J.Bernussou : L.A.A.S. du C.N.R.S. 7, Avenue du Colonel Roche, 31077 TOULOUSE CEDEX 4. Email : garcia@laas.fr, bernusou@laas.fr. * Also with I.N.S.A. de TOULOUSE, Complexe scientifique de Rangueil 31077 TOULOUSE CEDEX 4.

The paper is organised as follows. The following section recalls some basic facts on diffusive state space representation of integrodifferential operators and the corresponding finite dimensional approximations. section 3 presents the identification method with some of its possible extensions. Section 4 is devoted to the illustration of the method on a lead acid battery. We end the paper by some concluding remarks and prospectives.

## 2. Preliminaries

In this paragraph, we briefly recall the diffusive realisations of fractional integrodifferential operator. The interested reader is referred to [1] or [3] for more details. This kind of realisation was first introduced in [1] and used to solve or explain a wide variety of problems where the "fractional nature" is present and central [3], [2]. An example of such operator is given by the fractional integration defined as :

$$y = I^{\alpha} u, \tag{2.1}$$

where $\alpha > 0$ is the real order of integration, $u$ is the input and $y$ is the output. The Laplace transform of the convolution kernel associated to (2.1) is given by [3] :

$$\frac{1}{s^{\alpha}}. \tag{2.2}$$

When $\alpha \in ]0, 1[$, the impulse response of the operator can be written as :

$$y(t) = \int_0^{\infty} \mu(\xi) e^{-\xi t} d\xi, \tag{2.3}$$

with :

$$\mu(\xi) = \frac{sin \pi \alpha}{\pi} \xi^{-\alpha}. \tag{2.4}$$

Obviously from (2.3), such a dynamic system can be realised in the following way :

$$\left\{ \begin{array}{lcl} \partial_t X(\xi, t) & = & -\xi X(\xi, t) + u(t), \;\; X(\xi, 0) = 0 \\ y(t) & = & \int_0^{\infty} \mu(\xi) X(\xi, t) d\xi, \end{array} \right. \tag{2.5}$$

where $\xi \geq 0$ and the state $X$ belongs to an appropriate Hilbert space [3]. Note that (2.5) is an infinite dimension model. The diffusive nature of this model is straightforward noting its equivalency with the representation :

$$\left\{ \begin{array}{lcl} \partial_t \Phi & = & {\partial_x}^2 \Phi + u \otimes \delta, \;\; X(\xi, 0) = 0 \\ y(t) & = & \int_{\mathbb{R}} M \Phi dx, \end{array} \right. \tag{2.6}$$

In [3], it is shown that defining an appropriate space of distributions $\mu$, (2.5) can realize a wide class of operators with a Banach space structure, which is of great interest for optimisation problems (such as identification). This class includes the "integrodifferential operators" of order less than 1 [3].

Now for computational and practical reasons, it is often necessary to consider a finite dimensional approximation of (2.5). To obtain this approximated model, a finite number of values of $\xi$ are considered, say $\Xi = \{\xi_k\}_{k=1,N} \subset \mathbb{R}^+$ and a finite dimensional approximation of (2.5) can be written as :

$$\left\{ \begin{array}{lcl} \dfrac{dX_i(t)}{dt} & = & -\xi_i X_i(t) + u(t) \\ X_i(0) & = & 0, i = 1, \ldots, N \end{array} \right. \tag{2.7}$$

Note that $X_i(t) = X(\xi_i, t)$. The output approximation denoted by $\bar{y}(t)$ is obtained in two steps [2] :

- constructing a state approximation using interpolating functions $\Lambda_k$

$$\bar{X}(\xi,t) = \sum_{i=1}^{N} X_i(t)\Lambda_i(\xi) \tag{2.8}$$

- Obtaining $\bar{y}(t)$ by

$$\bar{y}(t) = \int_{\mathbb{R}+} \mu(\xi)\bar{X}(\xi,t)d\xi \tag{2.9}$$

Hence, writing the approximated model in a classical state equation form leads to :

$$\begin{cases} \dfrac{dX(t)}{dt} & = & AX(t) + Bu(t) \\ \bar{X}(t) & = & CX(t), \end{cases} \tag{2.10}$$

with

$$\begin{array}{l} X(t) = (X_1(t), ..., X_N(t))',\ X(0) = 0 \\ A = diag(-\xi_1, ..., \xi_N) \\ B = (1, \dots, 1)',\ C_i = \int_{\mathbb{R}+} \mu(\xi)\Lambda_i(\xi)d\xi,\ i = 1, \dots, N \end{array}$$

In [5], the convergence property of the approximation defined above is investigated. The following result may be proved.

LEMMA 2.1. *: By a convenient choice of the interpolating functions, for a given $u \in L^2(0,T)$ when $N \to \infty$ :*

$$\begin{array}{c} \|X - \bar{X}\|_{L^2(0,T;V_\alpha)} \to 0, \\ \|y - \bar{y}\|_{L^2(0,T)} \to 0, \end{array}$$

*where $V_\alpha$ is an appropriate Hilbert space for $X$ and $\bar{X}$.*

In [2] some choices for the interpolating functions and for the set $\Xi$ are proposed. In the numerical example discussed in this paper, a geometric sequence is considered. $N$ values of $\xi$ are taken in the interval $[\xi_{min}, \xi_{max}]$ such that $\xi_{k+1} = r\xi_k$, $r$ being equal to $(\xi_{min}/\xi_{max})^{1/(N-1)}$. This frequently used choice [7] produces a linear sequence of pulsations on a Bode plot and offers a good compromize between the width of the band of interest and the complexity of the obtained approximation.

## 3. Identification of a diffusive model

We consider the case where we only dispose of measurements (input, output) obtained from an experiment on a real system. If the system can be modelized by a diffusive equation such as (2.5), a way to deduce a model is to extract from the data, an estimation of $\mu$ by an identification procedure. We suppose that we dispose of measurements $\hat{u}$ and $\hat{y}$ on the interval of time $[T_i, T_f]$.

### 3.1. The infinite dimension framework

The objective is to identify an input/output diffusive model :

$$\begin{cases} \partial_t X(\xi,t) & = & -\xi X(\xi,t) + u(t),\ X(\xi,0) = 0 \\ y(t) & = & \int_0^\infty \mu(\xi)X(\xi,t)d\xi, \end{cases} \tag{3.1}$$

The main feature of the previous model is that structure of the state equation is fixed. The difference between two models is captured through the distribution $\mu$. In fact, among the equivalent realisations proposed in [2], (3.1) seems to be the more adequate for the purpose of identification, because $\mu$ appears linearly on the output of the model.

If we express the output through the convolutionproduct denoted by *, we obtain [3] :

$$y(t) = \int_{\mathbb{R}+} (e^{-t\xi} * u)\mu(\xi)d\xi$$

Associated to the previous expression, let us define the operator $K_u$ :

$$K_u : \begin{cases} \mathbf{M} \to L^2(T_i, T_f) \\ \mu \to y = K_u\mu \end{cases}, \tag{3.2}$$

$$(K_u\mu)(t) = \int_{\mathbb{R}+} (e^{-t\xi} * u)\mu(\xi)d\xi.$$

where $\mathbf{M}$ is a convenient Hilbert space of distributions. affects the data and diffusive model is not able to capture all the physical phenomena (nonlinearities,...). For these reasons, the identified model do not fit exactly the data. From a mathematical point of view, this means that it does not exist $\hat{\mu}$, solution of

$$\hat{y} = K_{\hat{u}}\hat{\mu}.$$

A way to solve the identification problem is to find a $\hat{\mu}$ which minimizes in some sense the error between $\hat{y}$ and $K_{\hat{u}}\hat{\mu}$, for example the distance :

$$||\hat{y} - K_{\hat{u}}\hat{\mu}||_{L^2(T_i,T_f)}.$$

It is well-known that the solution of this optimisation problem is given by :

$$\hat{\mu}(\xi) = (K_{\hat{u}}^* K_{\hat{u}})^{-1} K_{\hat{u}}\hat{y},$$

where $K_{\hat{u}}^*$ is the dual operator of $K_{\hat{u}}$ defined by :

$$\forall w \in L^2(T_i, T_f), \ < K_{\hat{u}}|w >_{L^2(T_i,T_f)} = < \mu|(K_{\hat{u}}^* w >_{\mathbf{M}} .$$

This solution is the best solution in the sense of this least square error. Note that other hilbert spaces could be chosen, for example Sobolev spaces $H^k(T_i, T_f)$ [8], for additional physical informations.

### 3.2. The finite dimension framework

Suppose that we approximate the model (3.1) choosing as described above, a finite number of values of $\xi$, $\Xi = \{\xi_k\}_{k=1,\ldots,N} \subset \mathbb{R}^+$. The approximate model reads :

$$\begin{array}{rcl} \dot{X}_i(t) & = & -\xi_i X_i(t) + u(t), \ i = 1, \ldots, N \\ Y_i(t) & = & \sum_{i=1}^N \mu_i X_i, \end{array} \tag{3.3}$$

Here, a remark is necessary to discuss this approximation with respect to the one defined by (2.7). In section 2, an approximation of the state $X(\xi, t)$ is performed by means of interpolating functions while in the model (3.3), an approximation of $\mu$ through a sum of Dirac masses $\{\delta_{\xi_i}(\xi), \ i = 1, \ldots, N\}$ is taken into account. For identification purpose, this approximation is the most appropriate for the construction of a finite dimensional model. It is then possible to build a $\xi$-continuous model by considering $\mu(\xi) = \sum \mu_i \Lambda_i(\xi)$, with $\Lambda_i$ replacing $\delta_{\xi_i}$. This allows the access to infinite dimension models from a wide variety of ways.

Now define the operator $K_{\hat{u}}$

$$\begin{array}{c} K_{\hat{u}} : \mathbb{R}^{\mathbf{N}} \longrightarrow L^2(T_i, T_f) \\ \hat{\mu} = (\ \hat{\mu}_1, \ldots, \hat{\mu}_N\ )' \to y(t) = \displaystyle\sum_{i=1}^N \hat{\mu}_i (e^{-t\xi_i} * u). \end{array} \tag{3.4}$$

The dual operator $K_{\hat{u}}^*$ is defined by :

$$K_{\hat{u}}^* : L^2(T_i, T_f) \longrightarrow \mathbb{R}^{\mathbf{N}}$$

$$\begin{aligned} < K_{\hat{u}}\hat{\mu}|y(t) >_{L^2(T_i,T_f)} &= \int_{T_i}^{T_f} \sum_{i=1}^{N} \hat{\mu}_i (e^{-t\xi_i} * u) y(t) dt \\ &= \sum_{i=1}^{N} \hat{\mu}_i \int_{T_i}^{T_f} (e^{-t\xi_i} * u) y(t) dt \\ &= < \hat{\mu}|K_{\hat{u}}^* y >_{\mathbb{R}^N} . \end{aligned} \tag{3.5}$$

Choosing for simplicity $\mathbf{M} = L^2(\mathbb{R}^+)$, the induced norm in $\mathbb{R}^N$ is the ordinary euclidian one and :

$$< \hat{\mu}|K_{\hat{u}}^* y >_{\mathbb{R}^N} = \sum_{i=1}^{N} \hat{\mu}_i (K_{\hat{u}}^* y)_i$$

which lead to :

$$(K_{\hat{u}}^* y)_i = \int_{T_i}^{T_f} (e^{-t\xi_i} * u) y(t) dt. \tag{3.6}$$

On the other hand, we have :

$$\begin{aligned} (K_{\hat{u}}^* K_{\hat{u}} \hat{\mu})_i &= \int_{T_i}^{T_f} (e^{-t\xi_i} * u) \sum_{j=1}^{N} \hat{\mu}_j (e^{-t\xi_j} * u) dt \\ &= \sum_{j=1}^{N} \int_{T_i}^{T_f} (e^{-t\xi_i} * u)(e^{-t\xi_j} * u) \hat{\mu}_j dt, \end{aligned}$$

and :

$$(K_{\hat{u}}^* K_{\hat{u}})_{ij} = \int_{T_i}^{T_f} (e^{-t\xi_i} * u)(e^{-t\xi_j} * u) dt,$$

the estimated measure $\hat{\mu}$ is given by :

$$\hat{\mu}(\xi) = (K_{\hat{u}}^* K_{\hat{u}})^{-1} K_{\hat{u}} \hat{y}.$$

Remark 3.1.

In the case where $u(t) = \delta(t)$, we have :

$$(K_{\hat{u}}^* y)_i = \int_{T_i}^{T_f} e^{-t\xi_i} y(t) dt$$

$$\begin{aligned} (K_{\hat{u}}^* K_{\hat{u}})_{ij} &= \int_{T_i}^{T_f} e^{-t(\xi_i+\xi_j)} * dt \\ &= \frac{e^{-T_i(\xi_i+\xi_j)} - e^{-T_f(\xi_i+\xi_j)}}{(\xi_i+\xi_j)}. \end{aligned}$$

which allows to compute $\mu$ from a given impulse response.

### 3.3. The discrete-time expression

In numerous practical cases, the signals are sampled. If $\Delta T$ is the sampling period, the discrete time approximated model is expressed as :

$$\begin{cases} X_i(n+1) &= e^{-\xi_i \Delta T} X_i(n) + \dfrac{1 - e^{-\xi_i \Delta T}}{\xi_i} u(n) \\ Y(n) &= \displaystyle\sum_{i=1}^{N} \mu_i X_i(n). \end{cases}$$

Some elementary calculations lead to :

$$X_i(n) = \sum_{l=1}^{N} \frac{e^{-\xi_i(l+n)}(e^{-\xi_i \Delta T - 1})}{\xi_i} u(l-1)$$

and

$$\begin{aligned} Y(n) &= (X_1(n), ..., X_N(n)) \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_N \end{pmatrix} \\ &= C(n)\mu, \end{aligned}$$

If we dispose of $M = \dfrac{T_f - T_i}{\Delta T}$ measurements for $u$ and $y$, $\hat{\mu}$ is obtained by :

$$\hat{\mu} = (H^* H)^{-1} H^* \hat{y},$$

where :

$$H = \begin{pmatrix} C(1) \\ \vdots \\ C(M) \end{pmatrix}.$$

The matrix $H$ can be computed using the following recursion formula :

$$\begin{cases} C_i(0) &= 0 \\ C_i(k) &= e^{-\xi_i \Delta T} C_i(k-1) + \dfrac{1 - e^{-\xi_i \Delta T}}{\xi_i} u(k-1) \\ & \quad i = 1, ..., N \; j = 1, ..., M \end{cases}$$

### 3.4. Important remarks

- For some systems (in particular for the battery), the output is affine (the sum of a constant and a combination of the states), that is :

  $$Y(n) = \sum_{i=1}^{N} \mu_i X_i(n) + Y_0.$$

  It is possible to determine the constant $Y_0$ by rewriting $Y(n)$ in the following way :

  $$Y(n) = (C(n)\ 1) \begin{pmatrix} \mu \\ Y_0 \end{pmatrix}'$$

  and $H$ becomes :

  $$H = \begin{pmatrix} C(1) & 1 \\ \vdots & \vdots \\ C(M) & 1 \end{pmatrix}.$$

- A same approach can be adopted for systems with a direct transmission. Then we have :

  $$\begin{aligned} Y(n) &= \sum_{i=1}^{N} \mu_i X_i(n) + Y_0 + Du(n) \\ &= (C(n)\ 1\ u(n)) \begin{pmatrix} \mu \\ Y_0 \\ D \end{pmatrix}, \end{aligned}$$

and :

$$H = \begin{pmatrix} C(1) & 1 & u(1) \\ \vdots & \vdots & \vdots \\ C(M) & 1 & u(M) \end{pmatrix}.$$

### 3.5. On line identification

In some cases, it is interesting to derive an algorithm which updates the measure $\hat{\mu}$ when a new measurement is available. When the system is slowly time varying, it allows to identify on line the parameter $\hat{\mu}$. Adopting a classical scheme [4], we obtain a recursive least square algorithm. If $H(k)$ denotes the $k^{th}$ row of $H$ and $\hat{\mu}(k)$ the $\hat{\mu}$ estmation at step $k$ respectively, we have :

$$\hat{\mu}(k) = \left(\sum_{i=1}^{k} H(i)^* H(i)\right)^{-1} \left(\sum_{i=1}^{k} H(i)'\hat{y}(i)\right).$$

Now defining :

$$P(k) = \left(\sum_{i=1}^{k} H(i)^* H(i)\right)^{-1},$$

and remarking that :

$$P^{-1}(k) = P^{-1}(k-1) + H(k)'H(k),$$

we obtain after some elementary calculations involving the matrix inversion lemma, the algorithm :

$$\begin{aligned} \hat{\mu}(k) &= \hat{\mu}(k-1) + \frac{P(k-1)H(k)^*}{1 + H(k)P(k-1)H(k)^*}(\hat{y}(k) - H(k)\hat{\mu}(k-1)) \\ P(k) &= P(k-1) - \frac{P(k-1)H(k)^*H(k)P(k-1)}{1 + H(k)P(k-1)H(k)^*} \\ P(0) &= C.I,\ C > 0 \text{ is a constant, } I \text{ is the identity matrix} \\ \hat{\mu}(0) &= 0 \end{aligned}$$

A convergence proof of this algorithm can be found in [6]

## 4. Application : the lead acid battery

We apply the proposed method to the identification of the dynamic model of a lead acid battery. The considered input is the current and output is the voltage. In fact, the obtained model is the impedance of the battery for a particular operational mode.

A lot of works has been done on the impedance study of lead acid batteries [12], [13], [14], [15]. In almost all these works, this study has been done using a transfer function analyser. It is well known that the phenomena behind the dynamic behavior of a battery are very complex and difficult to explain. In some of these works and particularly [14], the fractional character of the battery dynamic behavior was pointed out and explained through the fractal geometry of the relation which controls the interfacial energy and mass exchange [14].

Here the objective is to show, considering the battery as a "blackbox", how to derive a diffusive model which captures its dynamic behavior. The interpretation of the obtained results in terms of the underlying physical phenomena is not the aim here, but this aspect could be considered from an infinite dimension version of the identification model (namely under a heat equation representation). This is under study.

Two experiments are performed. The first one corresponds to a 50% discharge of the fully charged battery by constant intensity currents. Two cycles are considered. The discharge current is constant during 7 minutes for the first and two minutes for the second. Between the two cycles, the battery is quiescent during two hours. Figure 1 illustrates this experiment.

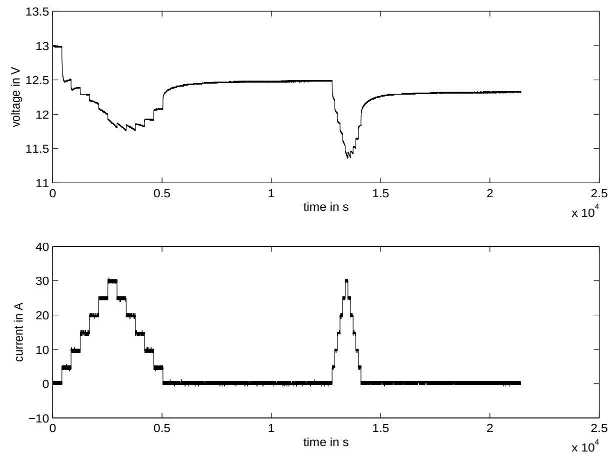


FIGURE 1. First experiment

In that experiment, the sampling period is equal to 1 s. In order to apply the identification method, we have to define the approximation set $\Xi$. It seems to be acceptable to take $\xi_{min} = 1/\Delta T$ and if we want to cover $D$ decades, we take $\xi_{max} = 10^D/\Delta T$. As mentionned in section 2, a geometric sequence of values of $\xi$ is considered. In the case of a battery, the output voltage varies around a constant voltage $Y_0$ (around 2 V per element). This constant will also beidentified by the identification procedure as explained in remark 3.1.

The results obtained taking 20 values of $\xi$ in $[\xi_{min}, \xi_{max}]$ with $D = 4$ are listed below.

$$\hat{\mu} = (0.0001,\ -0.0003,\ 0.0006,\ -0.0010,\ 0.0019,\ -0.0032,\\ 0.0056,\ -0.0105,\ 0.0194,\ -0.0315,\ 0.0440,\ -0.0575,\ 0.0768,\\ -0.1077,\ 0.1509,\ -0.1915,\ 0.2016,\ -0.1557,\ 0.0800,\ -0.0166)'$$

$$Y_0 = 12.8$$

The figure 2 depicted the corresponding Bode plot. Figure 3 shows how the identified model fits the data (the measured signal are also represented in this figure). In order to evaluate its validity, we test the model with measured data different to those which allowed the model identification. The comparison is given in figure 4. From the previous simulations, we observe a good agreement between the real system behavior and the one of the identified model which captures the dynamic behavior of the battery.

Another point appearing clearly is the fractional character of the model (see figure 2), in accordance with some physical studies and interpretation on the battery impedance [12]. In this case, the fractional order is around 0.25.

In the second experiment, we consider an initially fully charged battery, discharged by constant currents of about 10 A. Between the discharge phases, the

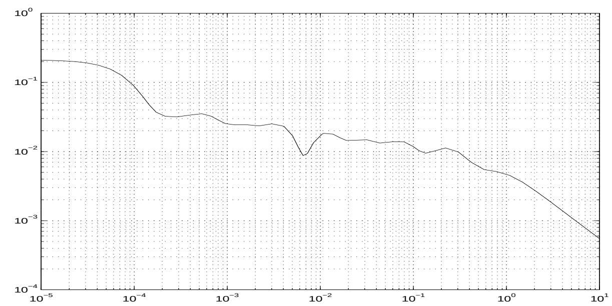


FIGURE 2. Bode plot of the identified diffusive model

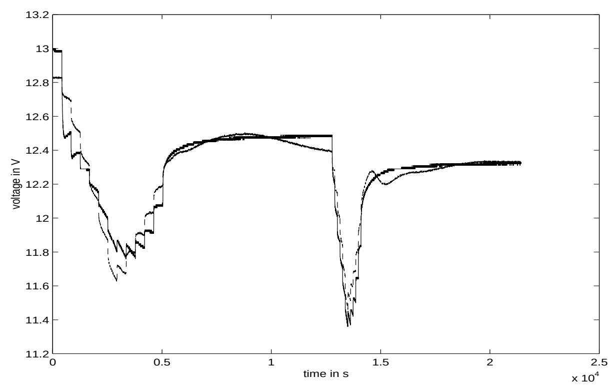


FIGURE 3. ”–” model ouput, ”-” system output”

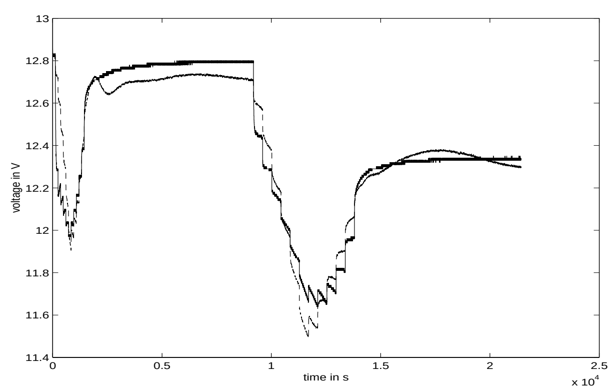


FIGURE 4. ”–” model ouput, ”-” system output”

battery is quiescent during about 2 hours. The sampling period is $\Delta T = 1s$. Figure 5 illustrates this experiment. As in the first experiment, we take 20 values of $\xi$ in $[\xi_{min}, \xi_{max}]$ and $D = 4$. We obtain :

$$\hat{\mu} = (0.0002,\ -0.0005,\ 0.0006,\ -0.0006,\ 0.0008,\ -0.0013,\\ 0.0022,\ -0.0036,\ 0.0059,\ -0.0099,\ 0.0163,\ -0.0262,\ 0.0403,\\ -0.0580,\ 0.0802,\ -0.0983,\ 0.1057,\ -0.0841,\ 0.0493,\ -0.0118)'$$

$$Y_0 = 12.8$$

The corresponding Bode plot is depicted in figure 6. A comparison between the

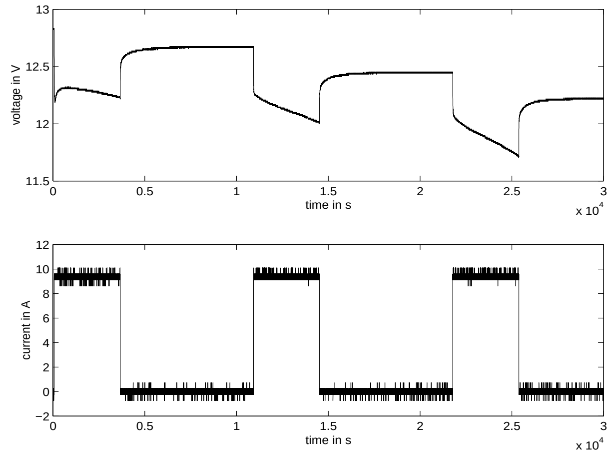


FIGURE 5. Second experiment

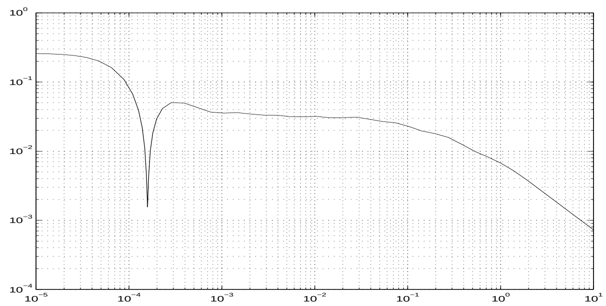

FIGURE 6. Bode plot

model output and the real output is presented in figure 7. Now, using this identified

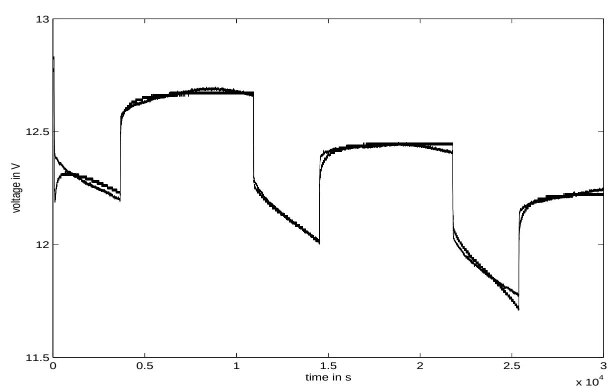


FIGURE 7. "–" model ouput, "-" system output"

model with the data of the first experiment, we obtain the curves in figure 8. The identified model does not fit so correctly the data of the first experiment. The values of currents vary between 0 A and 30 A while in the second experiment the current is maintained constant at 10 A. Roughly speaking, the system presents non negligible nonlinearities which can not be taken into account from our linear

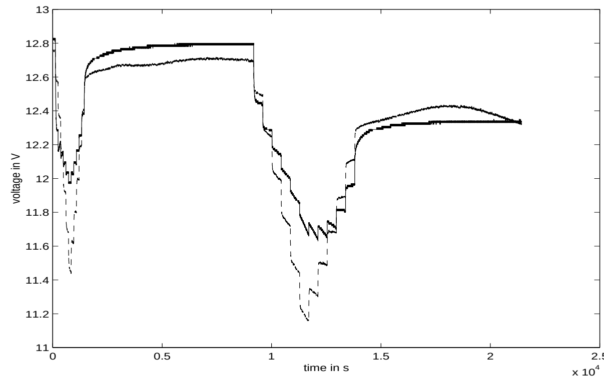


FIGURE 8. "–" model ouput, "-" system output"

approach. Note that the obtained model is valid around values of variables used for the identification.

## 5. Conclusion

We have presented a simple methodology to obtain a diffusive model from an input /output experiment on the system to be modelized. The crucial assumption is that the system behavior can be represented by a diffusive operator. Such systems or phenomena are frequently encountered in practice. The presented results must be considered as preliminar and a lot of problems have to be investigated more precisely. Among them we can extract the followings :

- In practice some noises affect the system and for modelling purpose, it is necessary to take them into account as done classically in identification problems. In this case, the effect of noise on the identified model has to be studied in detail.
- In the example treated in this paper, we can note that the estimation of $\mu$ alternates in sign. From a physical point of view, it seems to be natural that the function $\mu$ be positive. Indeed, in this case, the diffusive model possesses the passivity property and from a physical point of view, it is in conformity with the second principle of thermodynamics. In fact these problems are closely related to the mathematical framework and convergence problems in functional spaces. This suggests a redefinition of the identification problem in convenient hilbert spaces for $u$ and $y$ (Sobolev spaces) in place of $L^2(T_i, T_f)$ and $L^2(\mathbb{R}_\xi^+)$. This problem constitutes one of the major point to be investigated in a near future.
- Finally, the construction of an infinite dimension model could be of great interest for taking into account nonlinearities which are significant in PbO2/SO4 batteries

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