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A THREE DIMENSIONAL FINITE ELEMENT METHOD FOR A BIOLOGICAL ACTIVE SOFT TISSUE

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Abstract. We present a finite element method for large deformations response of an active incompressible nonlinear elastic and transversely isotropic soft tissue. The functional form of the strain energy function presented describes the mechanical properties of the active biological soft tissue and uses a time-dependent activation function. In addition we give the finite element equations for such a constitutive law. Assuming negligible body forces and inertia, a three-dimensional cylindrical element is used to obtain solution to an active contraction of a finite thick-wall anisotropic cylinder. The problem is chosen for the existence of an exact solution for the finite element model validation.

Résumé. Nous présentons une méthode d'éléments finis pour la simulation en élasticité non linéaire et en grandes déformations des déplacements d'un tissu musculaire actif, incompressible, transversalement isotrope. La fonction d'énergie présentée ici décrit les propriétés mécaniques du tissu actif et fait intervenir une fonction d'activation dépendant du temps. Supposant les forces internes et d'inertie négligeables, nous utilisons un élément cylindrique pour calculer les déplacements lors de la contraction active d'un cylindre fini à paroi épaisse et anisotrope. Les cas tests choisis permettent de valider la méthode grâce à la connaissance d'une solution analytique.

Introduction

Several numerical models, using finite element (FE) analysis, were proposed to simulate the heart continuously during the phases of the cardiac cycle [1–4]. In these previous studies, two approaches were used to model the living tissue. In both of them, the end-diastolic behavior of the muscle was derived from a passive strain-energy function expressed per unit of volume of the passive zero-stress state. Additionally, an active stress tensor was introduced to simulate the contraction of the biological tissue. The main limitation of the first modeling approach is that no active strain-energy function were used to obtain the active stress tensor, which suggests that the activated living tissue is not view as a continuum. In the second approach an active strain-energy function is introduced but an additional intuitive kinematics transformation of the zero-stress state is needed to derive the unloaded active state. This last point corresponds to the main limitation of this second modeling approach. Nevertheless Lin and Yin [6] proposed a continuum approach without any additional kinematics transformation but only for two specific states of the cardiac cycle (passive and maximal active states). Therefore, the purpose of this work was to propose a new method to model the material law of the living tissue, which avoids the previous limitations and allows to describe continuously the whole cardiac cycle. In addition, the finite element

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(FE) formulation with the proposed law was tested by considering simple cases, which are rectangular samples under different boundary conditions, as well as a finite thick-wall cylinder submitted to an internal pressure.

1. The mechanical model and its variational formulation

Constitutive law for the active artery tissue- To be consistent with our mathematical formulation, the letter $\mathbf{\Phi}$ is used for non elastic gradient tensor and the letter $\mathbf{F}$ is used for elastic gradient tensor.The activation of the muscle fibers changes the properties of the material and at the same time contracts the muscle itself. To have a continuous elastic description during the activation of the tissue, we used an approach similar to the one proposed by Ohayon and Chadwick [5], Taber [2], Lin and Yin [6]. From its passive zero-stress state P, the activation of the muscle fibers is modeled by two transformations (Fig. 1). The first one (from state P

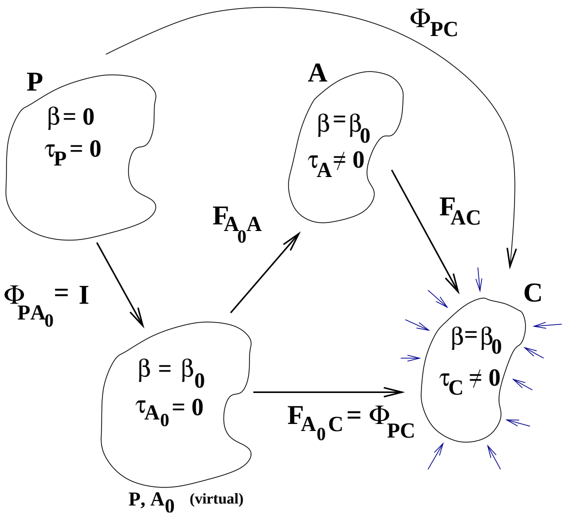

FIGURE 1. Description of the active rheology approach.

to virtual state A_0) changes the material properties without changing the geometry, and the second one (from A_0 to A) contracts the muscle without changing the properties of the material. Thus, the former is not an elastic deformation and is described by the gradient tensor $\mathbf{\Phi}_{PA_0} = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix. In that first transformation, only the strain energy function changing the rheology is modified using a time-dependent activation function $\beta(t)$ $(0 \leq \beta(t) \leq 1)$. The second transformation is an elastic deformation caused by the active tension delivered by the fibers and is described by the gradient tensor $\mathbf{F}_{A_0A}$. Finally, external loads are applied to state A deforming the body through $\mathbf{F}_{AC}$ into C. Thus the global transformation from state P to state C is a non elastic transformation ($\mathbf{\Phi}_{PC} = \mathbf{F}_{A_0C}\mathbf{\Phi}_{PA_0}$), but can be treated mathematically as an elastic one because $\mathbf{\Phi}_{PC} = \mathbf{F}_{A_0C}$. The change of the material properties during the activation is described by a time-dependent strain-energy function per unit volume of state P noted $W(\mathbf{E}_{PH}, t)$:

$$W(\mathbf{E}_{PH}, t) = W_{pas}(\mathbf{E}_{PH}) + \beta(t)W^{f}_{act}(\mathbf{E}_{PH}) \tag{1}$$

where $\mathbf{E}_{PH}$ is the Green's strain tensor at an arbitrary state H calculated from the zero strain state P (the state H could be one of the states A_0, A or C shown in figure 1), W_{pas} represents the contribution of the surrounding collagen matrix and of the passive fiber components, W^{f}_{act} arise from the active component of the embedded muscle fibers. The last term of the right side of the equation gives the variation of the mechanical muscle fibers properties during the activation. We treat the medium as a homogeneous, incompressible and hyperelastic material transversely isotropic with respect to the local muscle fiber direction. This last direction is characterized in an arbitrary state H by the unit vector $\mathbf{f}_H$. In this study, we modified the strain-energy function suggested by Lin and Yin [6] by substracting the beating term and introducing an activation function $\beta(t)$ which allows to describe continuously the phases of the cardiac cycle:

$$W_{pas}(\mathbf{E}_{PH}) = C_1^p(e^Q - 1) \tag{2}$$

$$\text{with} \qquad Q = C_2^p(I_1-3)^2 + C_3^p(I_1-3)(I_4-1) + C_4^p(I_4-1)^2 \tag{3}$$

$$\text{and} \qquad W_{act}^f(E_{PH}) = C_1^a(I_1-3)(I_4-1) + C_2^a(I_1-3)^2 + C_3^a(I_4-1)^2 + C_4^a(I_1-3) \tag{4}$$

where $(C_i^p, \quad i=1,\cdots,4)$ and $(C_i^a, \quad i=1,\cdots,4)$ are material constants and I_1, I_4 are two strain invariants given by $I_1(\mathbf{E}_{PH}) = tr\,\mathbf{C}_{PH}$ and $I_4(\mathbf{E}_{PH}) = \mathbf{f}_P \cdot \mathbf{C}_{PH}\mathbf{f}_P$ where $\mathbf{C}_{PH}$ is the right Cauchy-Green strain tensor ($\mathbf{C}_{PH} = 2\mathbf{E}_{PH} + \mathbf{I}$). Note that I_4 is directly related to the fiber extension λ_f ($I_4 = \lambda_f^2$).
To incorporate the active contraction, an active fiber stress $T^{(0)}$ was applied in the deformed fiber direction $\mathbf{f}_C$. Hence the Cauchy stress tensor in state C (noted τ_C) is given by

$$\boldsymbol{\tau}_C = -p_C\mathbf{I} + \boldsymbol{\Phi}_{PC}\frac{\partial W\left(\mathbf{E}_{PC},t\right)}{\partial \mathbf{E}_{PC}}\boldsymbol{\Phi}_{PC}^T + \beta(t)\,T^{(0)}\mathbf{f}_C\otimes\mathbf{f}_C \tag{5}$$

where p_C is the Lagrangian multiplier resulting of the incompressibility of the material, equivalent to an internal pressure, and the symbol $\otimes$ denotes the tensor product.

Variational formulation- The undeformed body state P contains a volume V bounded by a closed surface $\mathcal{A}$, and the deformed body state is, as before, noted C. The corresponding position vectors, in cartesian base unit vectors, are $\mathbf{R} = Y^R\mathbf{e}_R$ and $\mathbf{r} = y^r\mathbf{e}_r$, respectively. However, we write the equations with suitable curvilinear systems of world coordinates noted Θ^A in the reference configuration (state P) and θ^α in the deformed configuration (state C). In this paper we use the following conventional notations: (i) capital letters are used for coordinates and indices of tensor components associated to state P, and lower case letters are related to state C, and (ii) $\mathbf{G}$ and $\mathbf{g}$ are the base vectors in states P and C, respectively, for which parenthetical superscript indicates the associated coordinate system (for example $\mathbf{G}_I^{(x)} = \partial\mathbf{R}/\partial X^I = \mathbf{R}_{,I}^{(x)}$ and $\mathbf{g}_i^{(x)} = \partial\mathbf{r}/\partial x^i = \mathbf{r}_{,i}^{(x)}$).
The Lagrangian formulation of the virtual works principle is given by

$$\int_V P^{IJ}\Phi_J^{\cdot\alpha}\nabla_I(\delta u_\alpha)\,dV = \int_V \rho(b^\alpha-\gamma^\alpha)\delta u_\alpha\,dV + \int_{A_2}\mathbf{s}.\boldsymbol{\delta}\mathbf{u}\,dA \tag{6}$$

where P^{IJ} are the components of the second Piola-Kirchhoff stress tensor $\mathbf{P} = \boldsymbol{\Phi}_{PC}^{-1}\,.\,\boldsymbol{\tau}_C\,.\,(\boldsymbol{\Phi}_{PC}^{-1})^T$ referred to the base tensor $\mathbf{G}_I^{(x)}\otimes\mathbf{G}_J^{(x)}$, $\Phi_I^{\cdot\alpha} = \partial\theta^\alpha/\partial X^I$ are the components of the gradient tensor $\boldsymbol{\Phi}_{PC}$ in the base tensor $\mathbf{g}_\alpha^{(\theta)}\otimes\mathbf{G}^{(x)I}$, $\boldsymbol{\delta}\mathbf{u} = \delta u_\alpha\mathbf{g}^{(\theta)\alpha}$ is an arbitrary admissible displacement vector, $\nabla_I(\delta u_\alpha) = \partial\delta u_\alpha/\partial X^I - \mathbf{g}_{\alpha,I}^{(\theta)}\cdot\mathbf{g}^{(\theta)\beta}\delta u_\beta$ are the components of the covariant differentiation vector $\boldsymbol{\delta}\mathbf{u}$ in the base vectors $\mathbf{g}^{(\theta)\alpha}$ (i.e. $\nabla_I(\delta u) = \nabla_I(\delta u_\alpha)\mathbf{g}^{(\theta)\alpha}$). The previous differentiation is done with respect to the locally orthonormal body coordinates (X^I, $I=1,2,3$) which coincide with the local muscle fiber direction in state P. The material density in the undeformed body state P is ρ, $\mathbf{b} = b^\alpha\mathbf{g}_\alpha^{(\theta)}$ is the body force vector per unit mass, $\boldsymbol{\gamma} = \gamma^\alpha\mathbf{g}_\alpha^{(\theta)}$ is the acceleration vector, $\mathbf{s}$ is the surface traction per unit area of $\mathcal{A}$, and A_2 is the part of $\mathcal{A}$ not subject to displacement boundary conditions. The Lagrangian formulation for incompressibility is given by

$$\int_V \left(\det g_{IJ}^{(x)} - 1\right)\,q\,dV = 0 \tag{7}$$

where $g_{IJ}^{(x)}$ is the metric tensor and q is an arbitrary admissible pressure. Eqs.(6)-(7) represent the variational formulation of a system of nonlinear partial differential equations.

2. Preliminary results

We use a three dimensional finite element with Lagrange trilinear interpolation for the displacements and uniform pressure [7] to compute an approximate solution of Eqs.(6)-(7) on a rectanguler or cylindrical mesh, where we neglect the acceleration and body forces ($\mathbf{b} = 0$, $\gamma = 0$). At this time we compared the exact solution to the numerical one obtained for the cases of a free contraction, uniaxial and equibiaxial extension of a rectangular mesh. These comparisons show an error less than 10^{-12} in the L^2 norm. Figure 2 shows a good agreement between our proposed constitutive law and the experimental data for an equibiaxial test of a thin myocardial sample. Other simulations (not presented here) were done for an artery under physiological loading conditions, in which the fibers are uniformly oriented in orthoradial direction.
This numerical tool may be adapted for large scale problems such as modeling normal or pathological heart or artery.

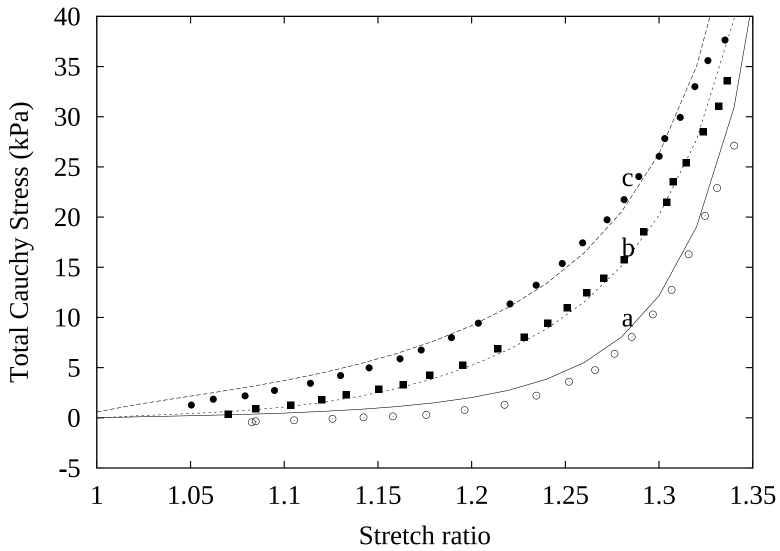

Figure 2. Equibiaxial tests: comparison with experimental data in passive case ($\beta = 0$, curve a) and in active cases ($\beta = 1$) in the fiber direction (curve c) and the cross-fiber direction (curve b). Solid lines represent the computed FE solutions, symbols represent the experimental data [6]

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