

REDUCED-ORDER REPRESENTATION OF TURBULENT JET FLOW AND ITS NOISE SOURCE

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Abstract. In the present study, subsonic turbulent jet noise is investigated employing reduced-order representations of the flow field and its noise source targeting 'least-order' approximations of the key processes. These representations utilize LES data for a compressible jet at Mach number 0.9 and Reynolds number 3600. The fluctuations of the velocity field and of the Lamb vector as noise source are investigated with three methods. Firstly, the streamwise development is characterised by a statistical analysis. Thus, the most active region of the flow field and the Lamb vector are observed at 11 and 8 jet diameters downstream, respectively. Secondly, an azimuthal mode decomposition is carried out. The first five azimuthal modes resolve most of the flow field and Lamb vector fluctuation. Thirdly, the dimension of the dynamics phase space is estimated by the proper orthogonal decomposition (POD). About 350 modes are necessary to resolve at least 50% of the fluctuation level of the hydrodynamics and even more modes are required for the noise source.

As expected, the end of the potential core correlates with the location of a distinct peak in the noise source magnitude, thus indicating a highly active acoustical region. Intriguingly, the noise source efficiency per unit energy increases with higher azimuthal modes. The comparison of the compressible jet results with the incompressible LES at the same Reynolds number reveals a significant smaller energy concentration in the first azimuthal modes, i.e. the incompressible flow is dynamically more complex. The current results are part of an ongoing effort to predict far-field jet noise by reduced-order modelling of its hydrodynamic source.

INTRODUCTION

The current study targets a coherent-structure based understanding of subsonic turbulent jet noise. Jet noise is the most important single noise source of mid- to large-scale civil aircrafts during takeoff. Increasing air traffic and more rigorous government regulations on traffic noise urge aero-engine producers to decrease the jet noise level. One key enabler to noise-reduction strategies is a good understanding of noise-producing mechanisms — in addition to experiments and simulations.

Coherent structures are known to be a cause of the noise, as noted already in Lighthill's classical paper in 1952 [1]. Indeed, the frequency of the local noise source scales approximately inversely with the size of the

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coherent structures [2]. That size increases in the streamwise direction by vortex merging, leading to a decrease of the dominant frequency. This relationship is experimentally corroborated and employed in surprisingly effective frequency slice models [3]. Direct numerical simulations [4] and large eddy simulations [5] [6] contribute to the understanding of experimental data [7].

The current models can explain, for instance, the effect of Zaman tabs on jet noise. The tabs induce a suppression of the fluctuation energy in the transition region by energising the mixing layer. This change is accompanied by a decrease of the dominant noise level at the expense of increasing the high-frequency level.

However, a more detailed aeroacoustic understanding of the unactuated and actively controlled jet is just recently evolving. Parabolised stability equations [8] can help to identify the noise sources of some compressible and heated jets. Low-dimensional vortex-filament models are proposed to explain low-Reynolds number jet noise [9]. Vortex-filament models of higher order reproduce high-Reynolds number jet noise [10] [11]. POD is proposed as one of the more systematic ways to identify noise-producing structures [12]. Yet, the lower bound for necessary POD modes is still estimated to be far above a hundred [13].

The current study is an endeavour towards a POD based reduced-order model of jet noise production. First (§1), the hybrid LES/CAA simulation is described. This simulation yields the hydrodynamics and noise source of a $Ma = 0.9$ jet at $Re = 3600$ adopting the parameters of [4]. Then (§2), various reduced-order representations are described. These filters distill the coherent structures and help to estimate the dimension of POD representations of the flow and noise source. The data analysis is detailed in §3. Finally (§4), these results are summarised and future efforts are outlined.

1. HYBRID LES / CAA JET SIMULATION

The hybrid LES/CAA method separates the wave propagation part from the sound generation part and exploits the disparity of the different scales between the turbulent and the acoustical field. A compressible flow simulation is performed via a large-eddy simulation enclosing the vicinity of the potential core and the spreading shear layers. In a subsequent step, the acoustical field is computed by solving the acoustic perturbation equations (APE) [14]. The analysis of the source-term of this set of equations provides an insight into the process of sound generation, i.e., those structures are extracted from the turbulent field generating the sound in the far field. A detailed investigation of these structures is the basis to understand the noise generation process in turbulent jets.

After describing the flow parameters, the governing equations, and the numerical procedure of the jet flow computation, the aeroacoustic analogy for the jet noise computation is resumed, i.e., the dominant source-term of this formulation is shown.

1.1. Jet configuration and flow computation

Let u_j and c_j be the jet nozzle exit velocity and sound speed, respectively, and T_j and T_∞ the temperature at the nozzle exit and in the ambient fluid. Then, a round turbulent jet at Mach $M_j = u_j/c_j = 0.9$ and Reynolds number $Re = 3,600$ at constant stagnation temperature T_0 and $T_j/T_\infty = 0.86$ is simulated. These flow parameters match those used in the experimental study by Stromberg et al. [15] and in the direct numerical simulation by Freund [4]. The governing equations based on the conservation of mass, momentum, and energy read

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot (\mathbf{F}^a - \mathbf{F}^d) = 0 \quad (1)$$

where $\mathbf{U}$ is the vector of the conserved quantities

$$\mathbf{U} = (\varrho, \varrho \mathbf{u}, \varrho E_t)^T. \quad (2)$$

ϱ is the density, $\varrho \mathbf{u}$ the specific momentum and ϱE_t the total specific energy. $\mathbf{F}$ represents the flux, where $\mathbf{F}^a$ is the advective part that contains the convective and the pressure terms and $\mathbf{F}^d$ is the diffusive part caused by

viscosity and heat conduction.

$$\mathbf{F}^a \begin{pmatrix} \rho \mathbf{u} \\ \rho \mathbf{u} \mathbf{u} + p \mathbf{I} \\ \mathbf{u}(\rho E_t + p) \end{pmatrix} \quad \mathbf{F}^d \begin{pmatrix} 0 \\ \boldsymbol{\tau} \\ \boldsymbol{\tau} \cdot \mathbf{u} - \mathbf{q} \end{pmatrix} . \quad (3)$$

To close the system the constitutive relationships for the stress tensor $\boldsymbol{\tau}$ by STOKES' hypothesis and for the heat-flux vector $\mathbf{q}$ by FOURIER'S law are added to this equation reading

$$\boldsymbol{\tau} = \eta(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3}\eta(\nabla \cdot \mathbf{u})\mathbf{I} \quad (4)$$

$$\mathbf{q} = -\kappa \nabla T \quad (5)$$

where η is the viscosity and κ is the heat conductivity. The fluid is assumed to be thermally and calorically perfect.

The compressible flow simulation uses the monotone-integrated LES approach based on the following approximations. A second-order accurate AUSM formulation is applied to the inviscid fluxes, while the diffusive fluxes are computed by a second-order central difference scheme. The time integration is performed by an explicit 5-stage Runge-Kutta scheme. More details can be found in Meinke et al. [16]. The inflow condition is specified by a hyperbolic-tangent profile onto which random fluctuations are superimposed, in order to excite instabilities in the jet [17]. For a proper simulation of the jet entrainment, a traction-free boundary condition [18] is implemented on the lateral boundaries, permitting inflow as well as outflow. The term "traction-free" means that the normal component of the stress tensor vanishes. The outflow boundary condition is based on a characteristics formulation. Furthermore, a damping zone is defined by an additional, stretched grid, in order to suppress possible reflections [19]. A sketch of the computational grid and the boundary conditions is shown in Fig. 1. The highlighted regions of Fig. 1 represents the inflow forcing at the inlet and the damping zone at the outlet.

1.2. Aeroacoustic analogy and noise source computation

Various acoustic equations are given in the literature. They mainly differ in the separation of the wave propagation part on the left hand side and the source-term formulation on the right hand side, which excites the acoustical field. The noise source which is investigated in the present paper results from the set of acoustic perturbation equations corresponding to the APE-4 formulation proposed by Ewert and Schröder [14] and includes, unlike the conventional wave equation, convection and refraction effects on the left hand side. It is derived by rewriting the conservation equations of mass and momentum for the density ρ , the pressure p , and the velocity $\mathbf{u}$ as

$$\frac{\partial p'}{\partial t} + \bar{c}^2 \nabla \cdot \left(\bar{\rho} \mathbf{u}' + \bar{\mathbf{u}} \frac{p'}{\bar{c}^2} \right) = \bar{c}^2 q_c \quad (6)$$

$$\frac{\partial \mathbf{u}'}{\partial t} + \nabla (\bar{\mathbf{u}} \cdot \mathbf{u}') + \nabla \left(\frac{p'}{\bar{\rho}} \right) = \mathbf{q}_m \quad (7)$$

with the right hand side sources

$$q_c = -\nabla \cdot (\rho' \mathbf{u}')' + \frac{\bar{\rho}}{c_p} \frac{\bar{D}s'}{Dt} \quad (8)$$

$$\mathbf{q}_m = -(\boldsymbol{\omega} \times \mathbf{u})' + T' \nabla \bar{s} - s' \nabla \bar{T} - \left(\nabla \frac{(u')^2}{2} \right)' + \left(\frac{\nabla \cdot \boldsymbol{\tau}}{\rho} \right)' . \quad (9)$$

The quantities c , s , T , $\boldsymbol{\omega}$ represent the sound velocity, the entropy, the temperature, and the vorticity vector. Moreover, the overbar denotes mean and the prime denotes perturbation values.

The dominant source-term of the APE system for cold jet noise has been shown in [20] to be the fluctuating Lamb vector, $\mathbf{L}' = (\boldsymbol{\omega} \times \mathbf{u})'$. This vorticity based source-term can be understood as a fluctuating vortex force, which is also part of the low Mach number acoustic analogies of Powell, Howe, and Möhring [21] [22] [23]. However, these acoustic formulations contain the divergence of the Lamb vector on the right hand side as the leading source-term for vortex sound generation.

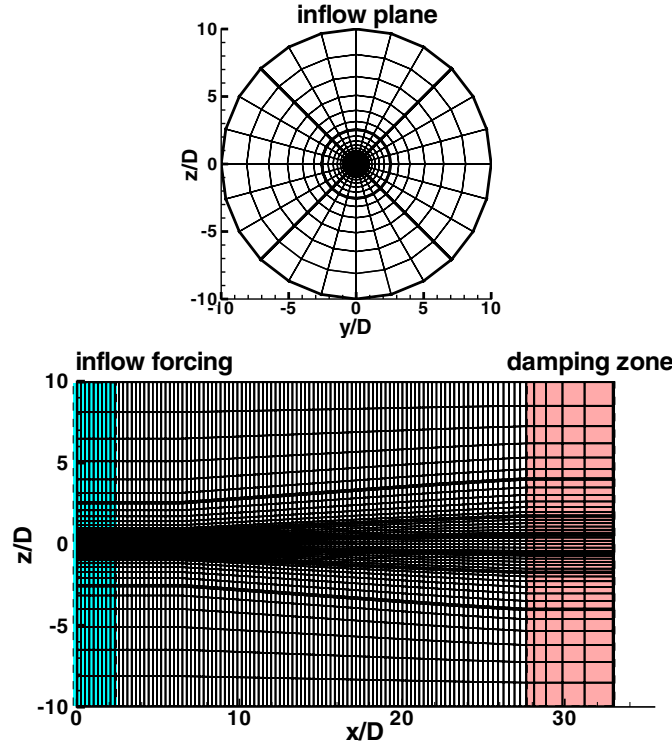


FIGURE 1. Jet configuration and computational grid. The grid is illustrated by selected lines.

2. REDUCED-ORDER ANALYSIS

In this section, the employed reduced-order analysis is described. We focus on the velocity field which characterises the hydrodynamics of the jet and the Lamb vector as the major noise source in the APE formalism. First (§2.1), the streamwise fluctuation level is quantified for the identification of the most active regions. Then (§2.2), the decomposition in azimuthal modes is recapitulated. The azimuthal contributions of the flow and the noise sources can be related to a variety of coherent structures with azimuthal symmetry in the literature. Finally (§2.2), the proper orthogonal decomposition (POD) methodology is described. The POD provides estimates for dynamical jet models and its farfield noise. The description focuses on the velocity field. The analysis of the Lamb vector is analogous. Fig. 2 provides a flow chart of the reduced-order analysis.

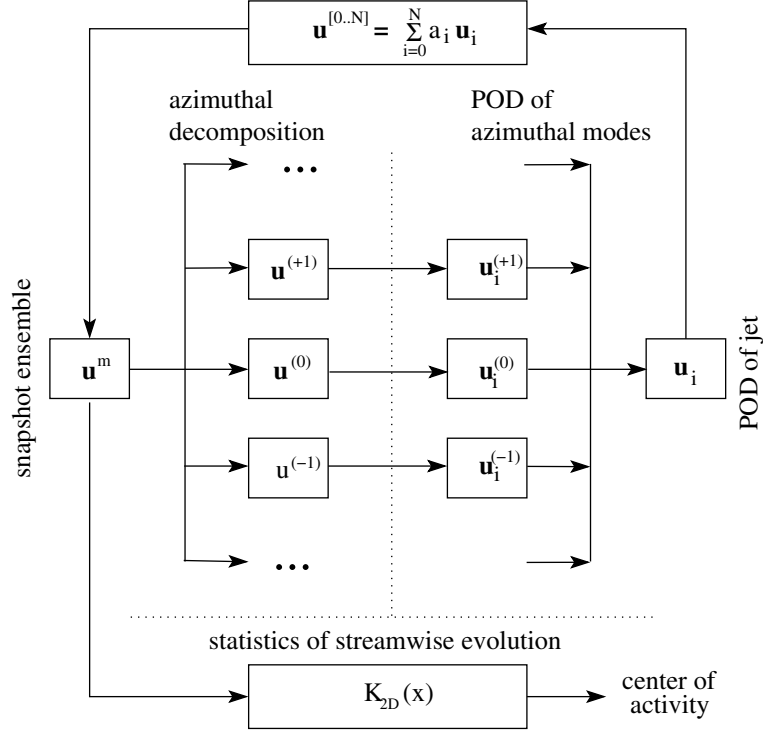


FIGURE 2. Reduced-order analysis of a snapshot ensemble. The azimuthal decomposition, the POD, and the statistics of the streamwise jet evolution is described in the text. Note that the azimuthal filter increases the effective number of snapshots and thus the number of POD modes. This is a conventional procedure exploiting the expected azimuthal invariance of the jet statistics [24].

2.1. Statistical analysis

The velocity field $\mathbf{u}$ is Reynolds decomposed in a time-averaged field $\mathbf{u}_0 := \overline{\mathbf{u}}$ and fluctuating contribution $\mathbf{u}'$,

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{u}_0(\mathbf{x}) + \mathbf{u}'(\mathbf{x}, t). \quad (10)$$

The fluctuation level¹ at a streamwise slice $x = \text{const}$ is quantified by

$$K_{2D}(x) := \frac{1}{2} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz \overline{\|\mathbf{u}'(x, y, z, t)\|^2}. \quad (11)$$

In practice, the transverse dimensions for integration are finite and chosen to approximate the infinite extent. The total fluctuation level in the domain $\Omega := \{(x, y, z) : 0 \leq x \leq x_{\max}\}$ is obtained by integrating over the streamwise distribution K_{2D} ,

$$K_{\Omega}(x) = \int_0^{x_{\max}} dx K_{2D}(x) = \frac{1}{2} \int_0^{x_{\max}} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz \overline{\|\mathbf{u}'(x, y, z, t)\|^2}. \quad (12)$$

¹For constant density, this term would correspond to the turbulent kinetic energy.

In complete analogy, the Lamb vector $\mathbf{L} = \mathbf{u} \times \boldsymbol{\omega}$ is Reynolds decomposed in $\mathbf{L}_0$ and $\mathbf{L}'$. Its fluctuation distribution and total level are denoted by $Q_{2D}(x)$ and Q_Ω , respectively.

2.2. Azimuthal decomposition

The azimuthal decomposition is based on the complete orthonormal set of trigonometric functions in $\mathcal{L}_2([0, 2\pi])$,

$$\Theta_l(\theta) := \begin{cases} \frac{1}{\sqrt{\pi}} \sin(l\theta) & \text{for } l > 0 \\ \frac{1}{\sqrt{\pi}} \cos(l\theta) & \text{for } l < 0 \\ \frac{1}{\sqrt{2\pi}} & \text{for } l = 0 \end{cases} \quad (13)$$

The decomposition of the velocity fluctuation in azimuthal modes $\mathbf{u}^{(l)}, l = 0, \pm 1, \dots$ is given by

$$\mathbf{u}'(\mathbf{x}, t) = \sum_{l=-\infty}^{\infty} \mathbf{u}^{(l)}(\mathbf{x}, t). \quad (14)$$

Here, the l -th term is described by the product ansatz $\mathbf{u}^{(l)}(\mathbf{x}, t) := \mathbf{f}^{(l)}(x, r, t) \Theta_l(\theta)$. We define azimuthal compound modes by $\mathbf{u}^{(\pm 0)} := \mathbf{u}^{(0)}$ and $\mathbf{u}^{(\pm l)} := \mathbf{u}^{(l)} + \mathbf{u}^{(-l)}$ for $l = 1, \dots, L$. The energy of the l -th azimuthal compound mode is defined by

$$K^{(\pm l)} := \frac{1}{2} \int_0^{x_{max}} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz \overline{\|\mathbf{u}^{(\pm l)}(\mathbf{x}, t)\|^2}. \quad (15)$$

It should be noted that the total fluctuation level is the sum of these azimuthal contributions, i.e. $K_\Omega = \sum_{l=0}^{\infty} K^{(\pm l)}$, due to orthogonality of the azimuthal modes with respect to the standard inner product

$$(\mathbf{f}, \mathbf{g})_\Omega := \int_0^{x_{max}} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz \mathbf{f} \cdot \mathbf{g}, \quad (16)$$

where $\mathbf{f}, \mathbf{g}$ are two vector fields defined in Ω .

The Lamb vector can be analysed in a completely analogous manner, replacing $\mathbf{u}$ by $\mathbf{L}$ and K by Q in equations (14) and (15).

2.3. Proper orthogonal decomposition

The LES data contains an ensemble of M instantaneous velocity fluctuations $\mathbf{v}^m := \mathbf{u}(\mathbf{x}, t_m) - \mathbf{u}_0(\mathbf{x})$, $m = 1, 2, \dots, M$ at different times t_m . The snapshot POD method extracts the most energetic POD modes $\mathbf{u}_i$ from that snapshot ensemble by suitable linear combinations of the fluctuations. The POD modes are orthonormal with respect to the inner product (16), i.e. $(\mathbf{u}_i, \mathbf{u}_j)_\Omega = \delta_{ij}$ for all $i, j > 0$. Details of the method are provided in [25] and [26].

Thus, an efficient reduced-order representation is obtained in the form of a Galerkin approximation with N modes,

$$\mathbf{u}'(\mathbf{x}, t) \approx \mathbf{u}^{[1 \dots N]}(\mathbf{x}, t) := \sum_{i=1}^N a_i(t) \mathbf{u}_i(\mathbf{x}), \quad (17)$$

where $a_i := (\mathbf{u}', \mathbf{u}_i)_\Omega$ are the time-dependent Fourier coefficients. The time-averaged energy content of the i -th modal contribution $\mathbf{u}^{[i]} := a_i \mathbf{u}_i$ is

$$K_i = \frac{1}{2} \overline{(\mathbf{u}^{[i]}, \mathbf{u}^{[i]})_\Omega} = \frac{1}{2} \overline{a_i^2} = \frac{1}{2} \lambda_i,$$

where λ_i is the i -th POD eigenvalue. The modes are sorted with respect to their energy content, i.e. $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots$. The expansion (17) has the optimality property, i.e., yields the smallest time-averaged residual with respect to the $\mathcal{L}_2$ -norm as compared to any other expansion with N modes.

The assumed rotation symmetry of the jet statistics implies that POD modes are Fourier modes with respect to the azimuthal angle. This mathematical result [26] is obeyed by experiment [7]. Moreover it can be utilized to simplify the POD computation by carrying out that decomposition on the snapshots after the azimuthal decomposition (Fig. 2). Thus, the 3D integrals of the correlation matrix become 2D integrals with respect to streamwise and radial coordinates.

3. RESULTS FOR TURBULENT JET FLOW AT $Ma = 0.9$ AND $Re = 3600$

In this section, reduced-order representations are extracted for the velocity field and the Lamb vector. The azimuthal decomposition and the POD method are employed on ensembles containing $M = 725$ snapshots for the velocity fluctuations and the Lamb vector fluctuations. Both ensembles are obtained by the hybrid LES/CAA jet simulation described in §2.

3.1. Reynolds decomposition

The Reynolds decomposition in mean flow and fluctuations is exposed for the first snapshot in Fig. 3. The snapshots of velocity and fluctuations represent coherent structures of the entrainment processes up to $x/D = 8$ followed by a strongly irregular behaviour. The visualisation of the mean flow emphasizes the jet expansion and the downstream deceleration of the streamwise component.

Fig.4 displays instantaneous contour plots of the flow field (left) and the perturbed Lamb vector and the divergence of the Lamb vector (right) in the plane $z = 0$. Four diameters downstream from the jet nozzle, coherent vortex structures become visible in the developing shear layers. Near the end of the potential core at $x/D = 8$ these structures suddenly break up. This mixing region is acoustically highly active as indicated by the increasing strength of the transverse components of the perturbed Lamb vector. Fluid from the ambient is entrained penetrating to the jet axis, while the fluid velocity of the jet flow decreases. The acceleration and deceleration of vortical structures near the end of the potential core might contribute to the noise generation process.

3.2. Statistical analysis

In this subsection, statistics of the flow and the acoustic field are analysed.

In Fig. 5 the mean center-line velocity shows an excellent agreement with the DNS solution by Freund [4] indicating a potential core length of approximately 7.9 jet diameters. More details on the statistics with respect to numerical and experimental results can be found in [20] showing that the computed relevant properties of the low Reynolds number jet are accurately captured. The pressure distribution along the centre-line foreshadows the entrainment process. Slow fluid from the ambient is entrained by the jet flow causing a jet velocity decrease which is accompanied by an increase in pressure. At the end of the potential core low density fluid from the ambient penetrates the jet axis mixing with the high density fluid from the jet.

Fig. 6 displays the fluctuation energy K_{2D} in the hydrodynamic field (left) and the fluctuation level Q_{2D} in the aeroacoustic field for streamwise slices according to Eqn. 11. Both curves show a distinct peak level, but at different axial positions. The hydrodynamic fluctuation energy reaches its peak value at an axial position $x/D \approx 12$, whereas the noise source fluctuation is maximised at the end of the potential core at $x/D = 7.9$. Furthermore, the peak of the noise source fluctuation is more pronounced than that of the hydrodynamic field and the curve drops off faster towards the end of the domain. This result indicates a bounded spatial extension of the acoustically active region, whereas the kinetic energy level of the hydrodynamic field is still high in an acoustically silent region.

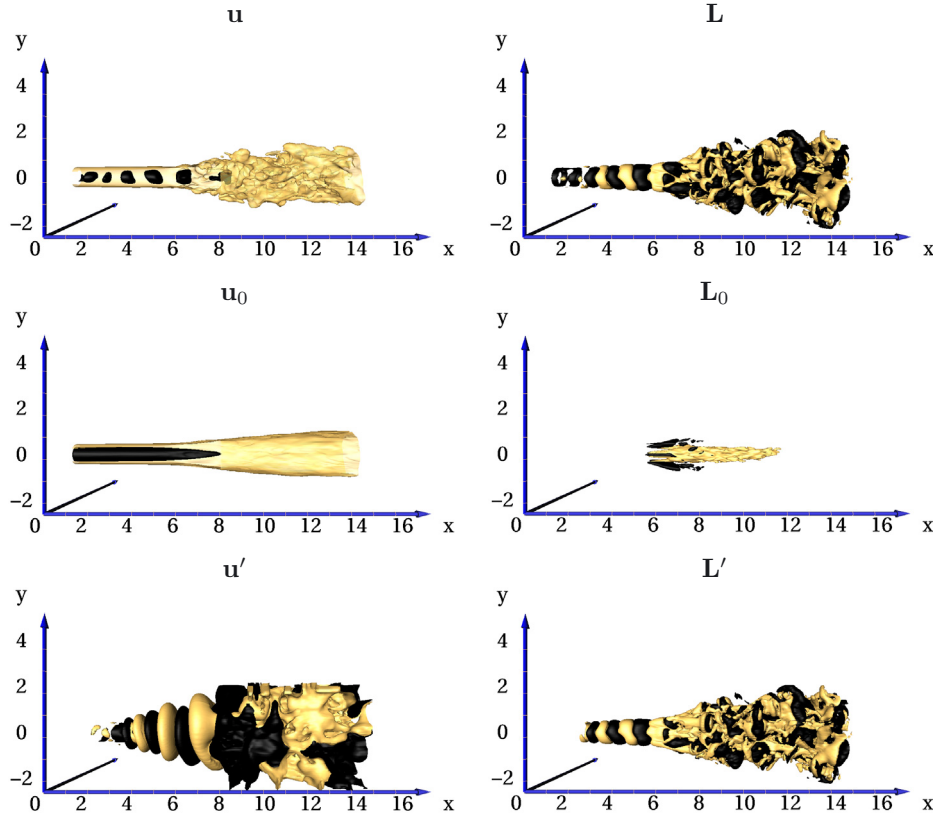


FIGURE 3. Snapshot of the velocity field (left) and the Lamb vector (right). From top to bottom, the instantaneous flow, the averaged flow, and the fluctuations are displayed. The field is visualised by isosurfaces of the x -component. The bright and dark isosurfaces represent the values 0.835 and 0.2 for the instantaneous flow, 0.8 and 0.2 for the averaged flow, and 0.005 and -0.005 for the fluctuations.

3.3. Azimuthal mode decomposition

In this section a snapshot is exemplarily decomposed in azimuthal modes. Moreover the resolution of the kinetic energy and of the noise source is shown for the first azimuthal compound modes.

The azimuthal modes $\mathbf{u}^{(l)}$ for $l = -3, \dots, 4$ of the velocity snapshot and the Lamb vector snapshot of Fig. 3 are shown in Figs. 7 and 8. The azimuthal modes $l = 0, -1, -2, -3$ of the cosine type in Fig. 7 are ordered from top to bottom by the wave number of the corresponding Θ_l . On the same line the azimuthal modes $l = 1, \dots, 4$ of the sine type are displayed in Fig. 8.

The azimuthal modes of Figs. 7 and 8 reproduce the structures of the entrainment processes from $x/D = 4$ to $x/D = 8$. In consistency with Fig. 6 the fluctuation structures of the azimuthal modes expand and become more irregular for $x/D > 8$.

The azimuthal modes $\mathbf{u}^{(l)}$, $l = -5, \dots, 5$ and therefore the azimuthal compound modes $\mathbf{u}^{(\pm l)}$, $l = 0, \dots, 5$ of the velocity fluctuations resolve 76.85% of the total kinetic energy K_Ω . The portions of the resolved total kinetic energy per compound mode $K^{(\pm l)}$ are displayed in the Fig. 9. Moreover, the portion of the resolved noise sources $Q^{(\pm l)}$ per compound mode $\mathbf{L}^{(\pm l)}$ of the Lamb vector fluctuations are also shown in Fig. 9. The

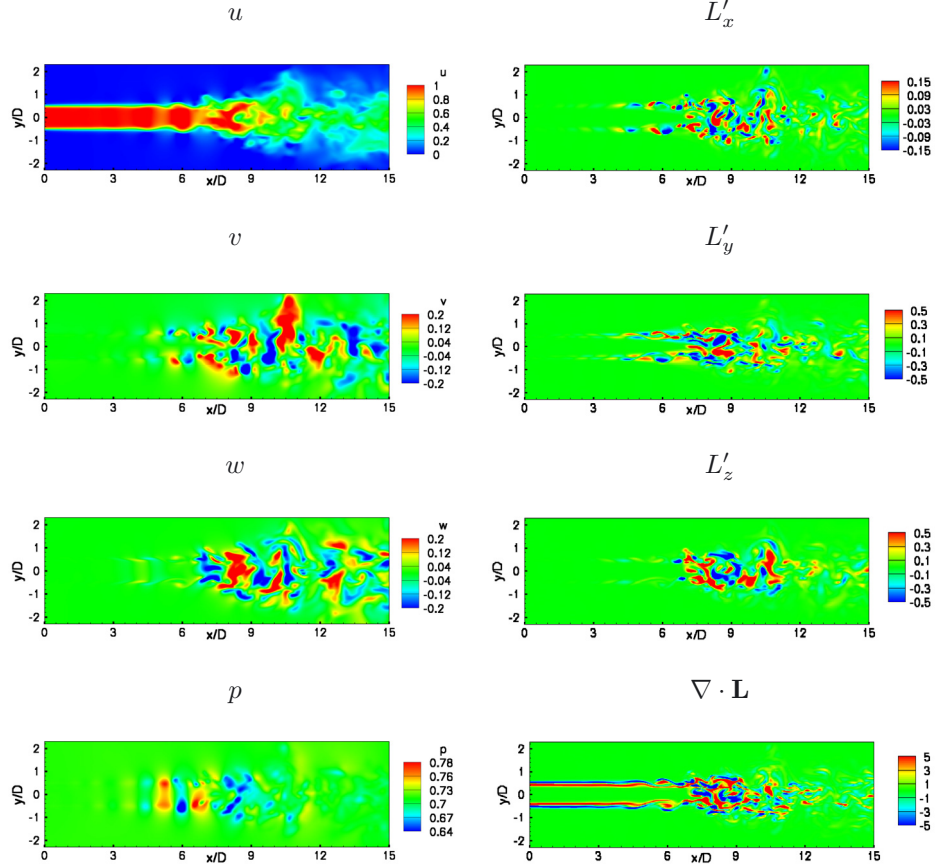


FIGURE 4. Same snapshot as figure 3, but displaying scalar fields of the velocity components and the pressure (left) and the components of the Lamb vector and its divergence (right) in the plane $z = 0$.

azimuthal compound modes $\mathbf{L}^{(\pm l)}$, $l = 0, \dots, 5$ and therefore the azimuthal modes $\mathbf{L}^{(l)}$, $l = -5, \dots, 5$ resolve 71.3% of the noise source Q_Ω .

In Fig. 10 the noise source efficiency $q^{(\pm l)} = Q^{(\pm l)} / K^{(\pm l)}$ is displayed. $q^{(\pm l)}$ increase monotonously for $l \geq 2$. Hence, for the purpose of understanding and controlling noise generation, the subsequent results have to be extended to modes with azimuthal order larger than 5. This hypothesis is supported by the slow decrease in $K^{(\pm l)}$ and $Q^{(\pm l)}$ for $l > 2$ in Fig. 9.

3.4. Proper orthogonal decomposition

The azimuthal decomposition of the snapshot ensembles with $M = 725$ provides ensembles of azimuthal modes $\{\mathbf{u}_m^{(l)}\}_{m=1}^M$. The POD eigenvalues λ_n^l , $n = 1, 2, \dots, M$, and their corresponding eigenmodes are generated by applying POD to those ensembles for each $l = -5, \dots, 5$. The $11 \cdot M = 7975$ eigenvalues λ_i , $i = 1, \dots, 11 \cdot M$,

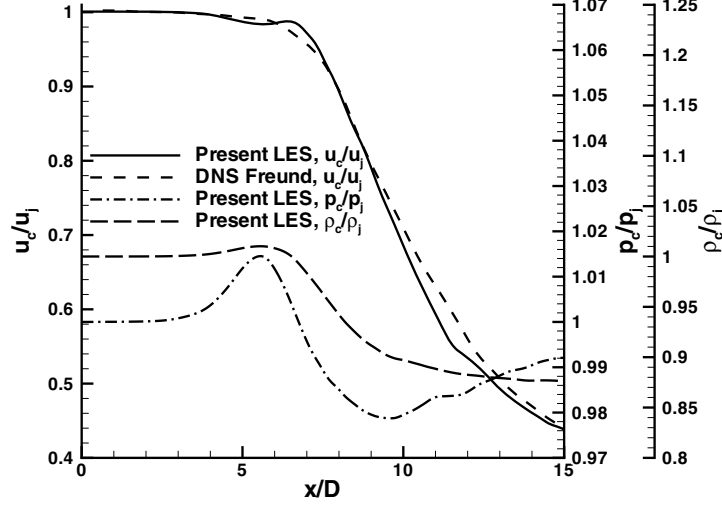


FIGURE 5. Characterisation of the mean flow. Normalised centre-line velocity u_c/u_j , density ρ/ρ_j and pressure distribution p/p_j as a function of the streamwise coordinate x .

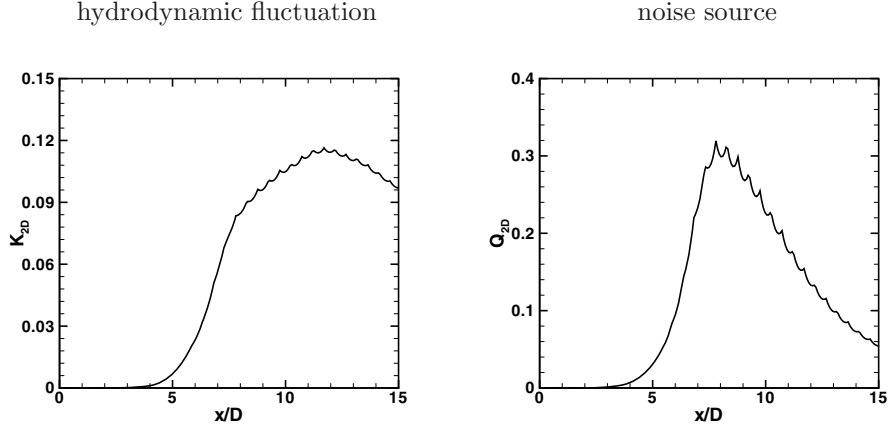


FIGURE 6. Streamwise distribution of the fluctuation energy $K_{2D}(x)$ (left) and of the noise source $Q_{2D}(x)$ (right).

are collected in a single decomposition and sorted by magnitude $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots$. The corresponding POD eigenmodes of the velocity fluctuations and the Lamb vector fluctuations are denoted by $\mathbf{u}_i$ and $\mathbf{L}_i$, respectively. The fluctuation field $\mathbf{u}'$ is approximated by the resulting reduced-order representation (17). For the Lamb vector field, that approximation $\mathbf{L}^{[1 \dots N]}$ is defined analogously.

In Fig. 11 the eigenvalues λ_n^l , $n = 1, \dots, M$ of the POD of the velocity fluctuations and of the POD of the Lamb vector fluctuations are displayed over the azimuthal index l . The eigenvalues of the POD of the velocity fluctuations are proportional to the portion of the kinetic energy resolved by the corresponding eigenmodes and these of the Lamb vector fluctuations are proportional to the noise source. Hence, the eigenmode resolving

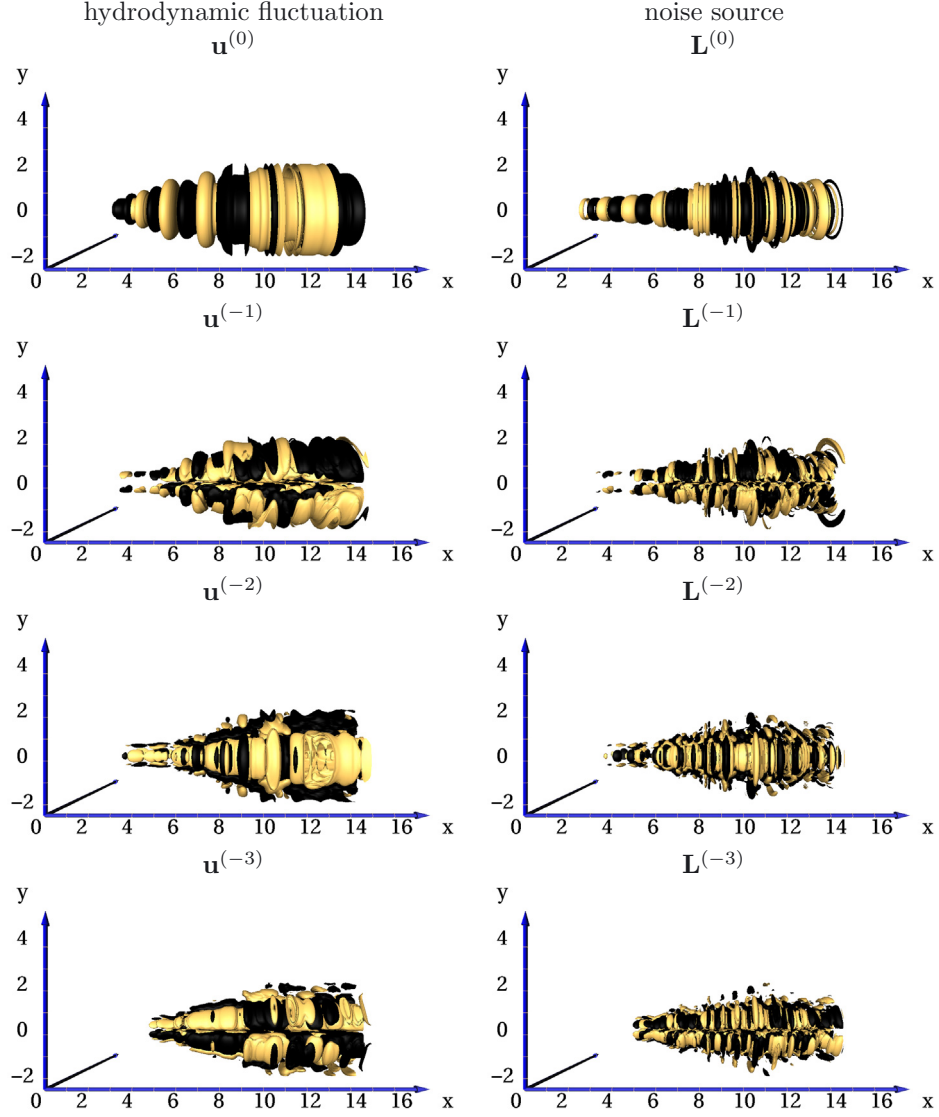


FIGURE 7. Azimuthal decomposition of an instantaneous fluctuation of the velocity $\mathbf{u}'$ (left) and the Lamb vector $\mathbf{L}'$ (right) employing the flow data of Fig. 3. From top to bottom, the figure displays the first azimuthal modes of the cosine type associated with the snapshot fluctuations. The fields are visualised like the fluctuations in Fig. 3.

the most kinetic energy is of the $\sin(2\theta)$ type, whereas the eigenmode resolving the most noise source is of the $\sin(1\theta)$ type. The close symmetry of both spectrograms refers to an energy equilibrium of the sine type with the corresponding cosine type azimuthal modes. This supports the hypothesis of stochastic stationarity of the snapshot ensemble in the azimuthal direction.

The eigenvalues λ_i decrease exponentially with the index i . The top row of Fig. 12 shows that decay for the velocity fluctuations and the Lamb vector fluctuations. In Fig. 13 the POD modes $\mathbf{u}_i$ and $\mathbf{L}_i$ are visualised.

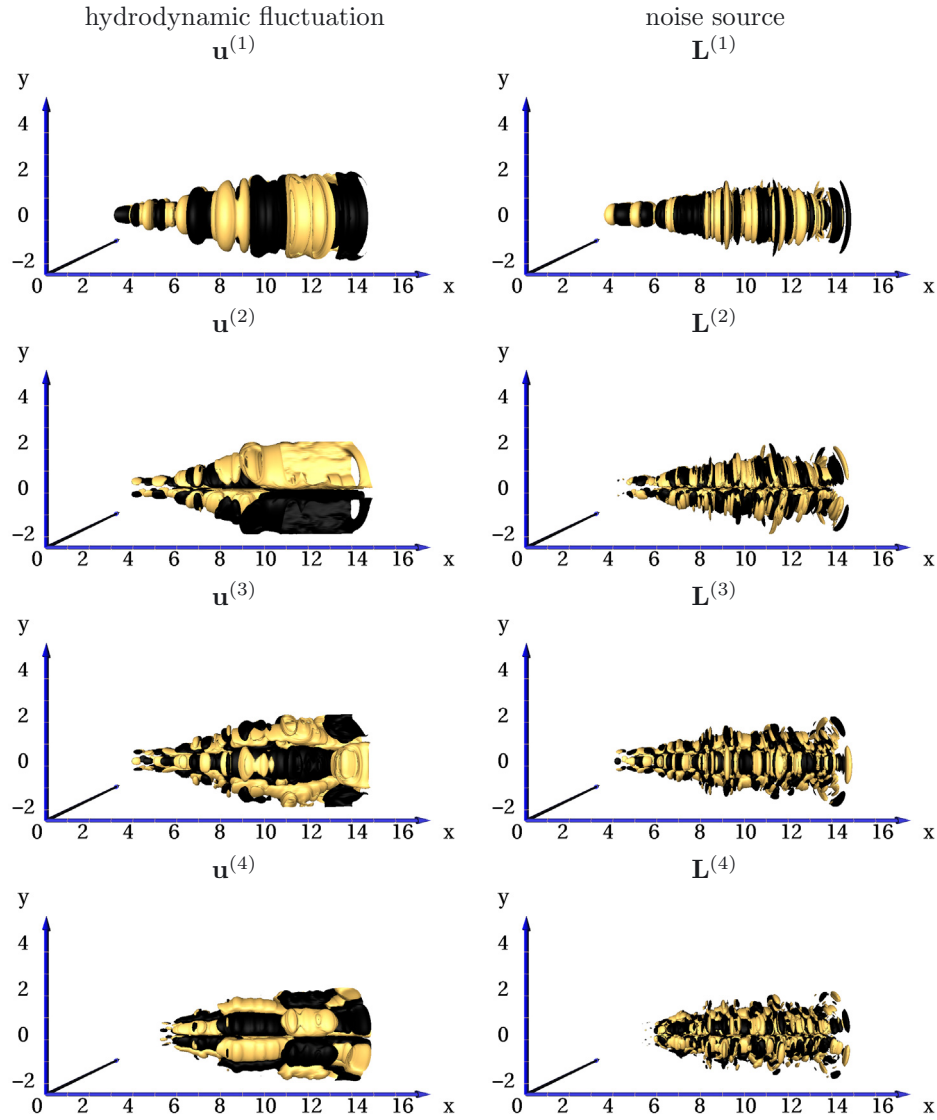


FIGURE 8. Azimuthal decomposition of an instantaneous fluctuation of the velocity $\mathbf{u}'$ (left) and the Lamb vector $\mathbf{L}'$ (right) employing the flow data of Fig. 3. The sine type azimuthal modes for $l = 1, \dots, 4$ are visualised like in Fig. 7. The velocities in the streamwise direction are equal to zero in the $z = 0$ plane for all azimuthal modes, which can not be seen in the given perspective.

In the bottom row of Fig. 12 the unresolved energies of the reduced-order representation $\mathbf{u}^{[1\dots N]}$ and $\mathbf{L}^{[1\dots N]}$ are displayed over N . These unresolved kinetic energies converge slowly to zero, so a large number of POD modes is necessary to ensure a high level of resolved energy.

A large portion of energy, however, is contained in the stochastic fluctuations. Therefore, the coherent structures can be reproduced on a low energy level by a reduced-order representation. This effect is demonstrated in Fig. 14. The reduced-order representation $\mathbf{u}^{[1\dots 350]}$ reproduces the structures of the entrainment processes from $x/D = 4$ to $x/D = 8$. The reduced-order representation $\mathbf{L}^{[1\dots 350]}$ reproduces coherent structures from

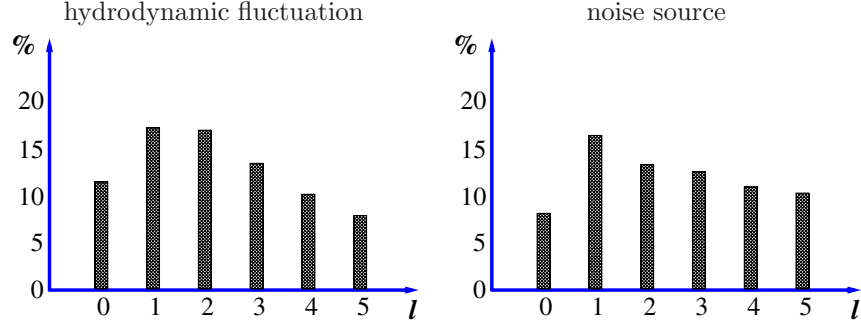


FIGURE 9. Azimuthal decomposition of the fluctuation energy $K^{(\pm l)}$ (left) per compound mode $\mathbf{u}^{(\pm l)}$ and of the noise source $Q^{(\pm l)}$ (right) per compound mode $\mathbf{L}^{(\pm l)}$. In the histogram $K^{(\pm l)}$ and $Q^{(\pm l)}$ are displayed as percentage of the total values K_Ω and Q_Ω over the azimuthal order l .

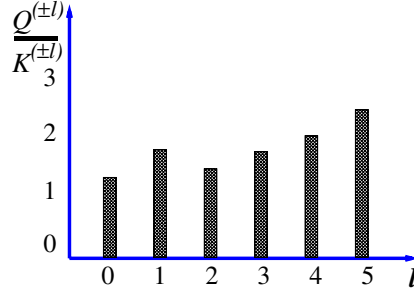


FIGURE 10. Azimuthal noise source efficiencies $q^{(\pm l)} = Q^{(\pm l)} / K^{(\pm l)}$ as a function of the azimuthal order l . Unlike in Fig.9, total values are considered.

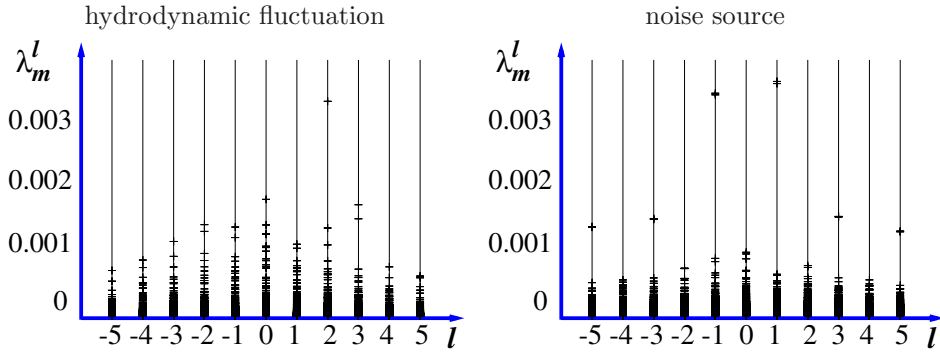


FIGURE 11. Azimuthal POD spectrogram of the velocity (left) and the Lamb vector (right). Each value of λ_n^l is indicated by a bar over the azimuthal order l .

$x/D = 0$ to $x/D = 6$. $\mathbf{u}^{[1...350]}$ resolves 50% of the total kinetic energy K_Ω and $\mathbf{L}^{[1...350]}$ resolves 27% of the noise source Q_Ω .

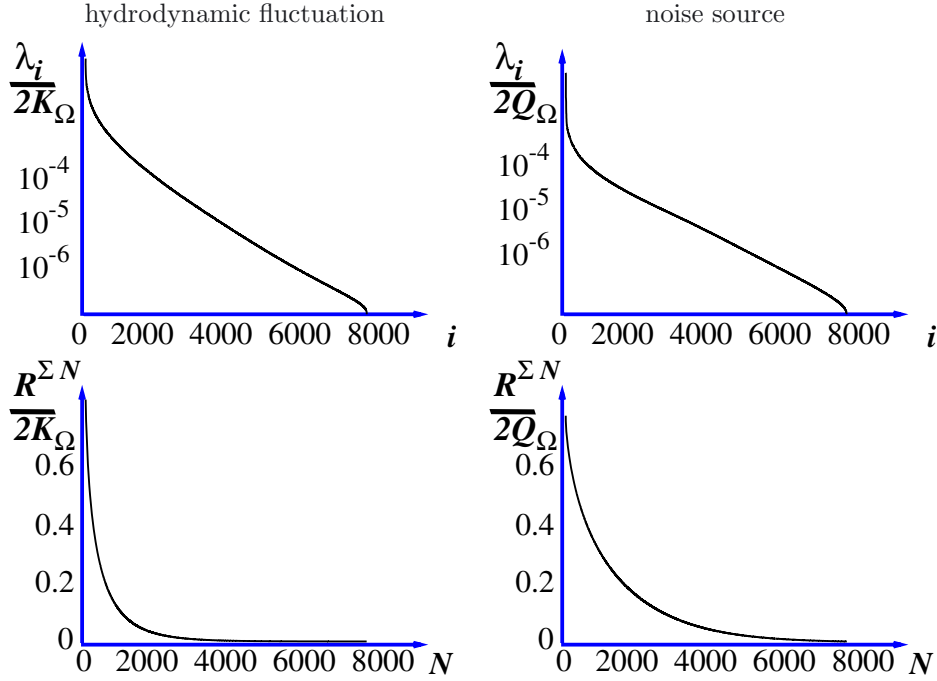


FIGURE 12. POD spectrum of the velocity (left) and the Lamb vector (right). The POD eigenvalues λ_i are normalised with the total fluctuation level (top row). For the y-axis a logarithmic scale is employed. In the same manner like the eigenvalues the unresolved energy $R^{\Sigma N} := R_s - \sum_{i=1}^N \lambda_i$ is normalised (bottom row). R_s is defined by $R_s := \sum_{l=0}^5 K^{(\pm l)}(Q^{(\pm l)})$, which is the maximum kinetic energy (noise source) having been resolved by the POD modes of the employed azimuthal modes. The gap $K_\Omega - R_s/2$ represents the unresolved kinetic energy of higher azimuthal modes. In the previous subsection the value of R_s is found to be 77% (71%) of the total fluctuation level of the velocity (noise source).

3.5. Comparison of compressible vs. incompressible jet flow

For a comparison between the results of compressible and incompressible flow, the reduced-order analysis is employed to a $M = 468$ velocity snapshot ensemble of the incompressible jet of [27] at the same Reynolds number 3600.

The azimuthal decomposition in compound modes of order $l = 0, \dots, 5$ is utilised in Fig. 15. As remarkable difference to the compressible flow, only 31.69% of K_Ω are resolved by these azimuthal compound modes.

By the combined azimuthal and POD analysis, described in the last subsection, $11 \cdot M \approx 5000$ eigenvalues are determined. These eigenvalues and the resolution of the associated reduced-order representation are obtained in Fig. 16. As for the compressible flow the resolution converges slowly to zero.

In conclusion, representation of velocity in incompressible flow requires significantly higher dimensions than compressible flow at a given energy resolution level.

4. CONCLUSIONS AND OUTLOOK

We investigated reduced-order representations targeting better understanding of turbulent jet noise. The flow has been computed by a LES at $Re = 3600$ and $Ma = 0.9$. The flow field is monitored by the fluctuation of

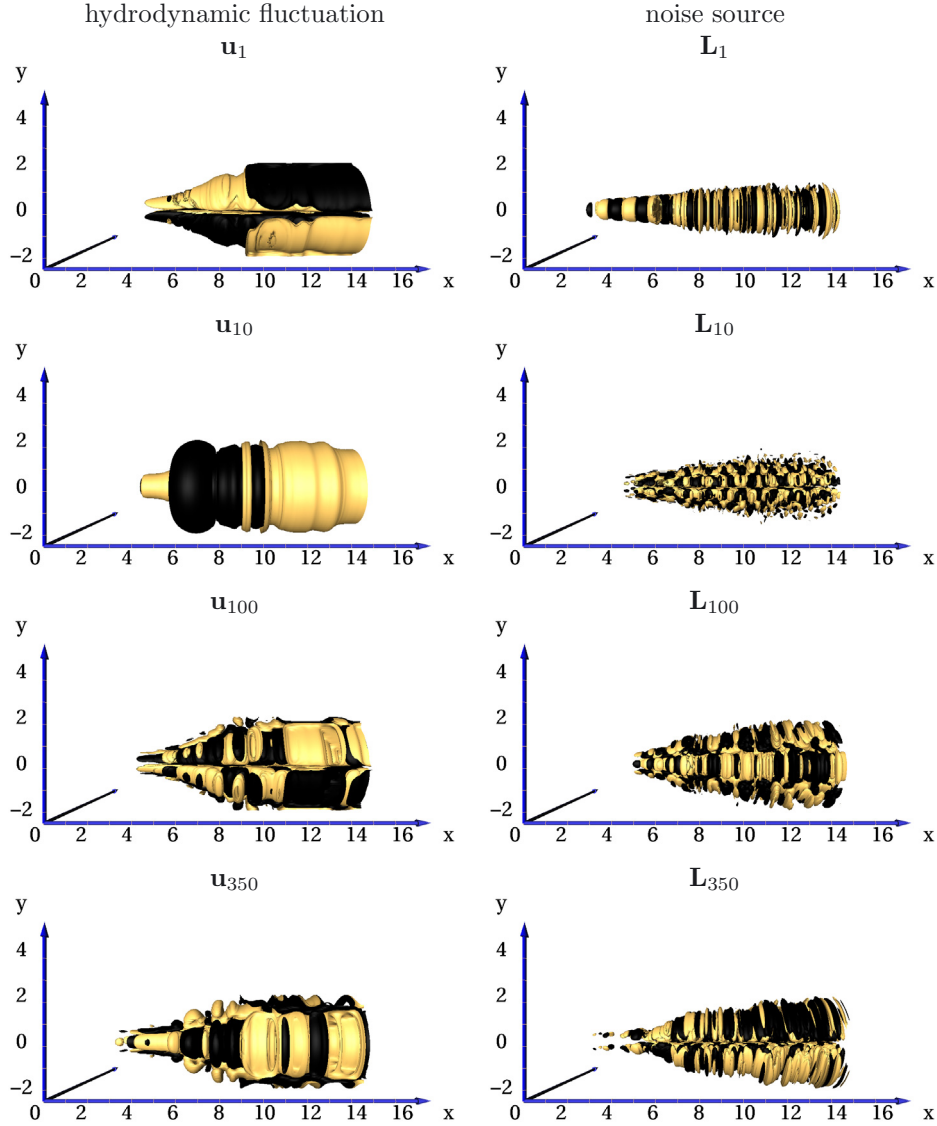


FIGURE 13. POD modes of the velocity field (left) and the Lamb vector (right). The subfigures display the POD modes $\mathbf{u}_i$ and $\mathbf{L}_i$ with indices $i = 1, 10, 100, 350$ from top to bottom. Vector fields are visualised as in Fig. 3.

the velocity. As noise source, the fluctuation of the Lamb vector is chosen. That vector constitutes the major acoustic source-term in the acoustic perturbation equations [14].

Three decompositions are employed for the flow field and the noise source. Firstly, the fluctuation level density in streamwise slices is determined. Thus, the noise source peak is observed 8 diameters downstream from the jet orifice. The velocity fluctuation yields its maximum between 11 and 12 diameters. The location of the noise source peak is consistent with the experimental investigation by Hileman et al. [7]. In a second step, an azimuthal decomposition is carried out. Finally, the azimuthally decomposed fields are analysed by POD to extract the most energetic structures in the jet.

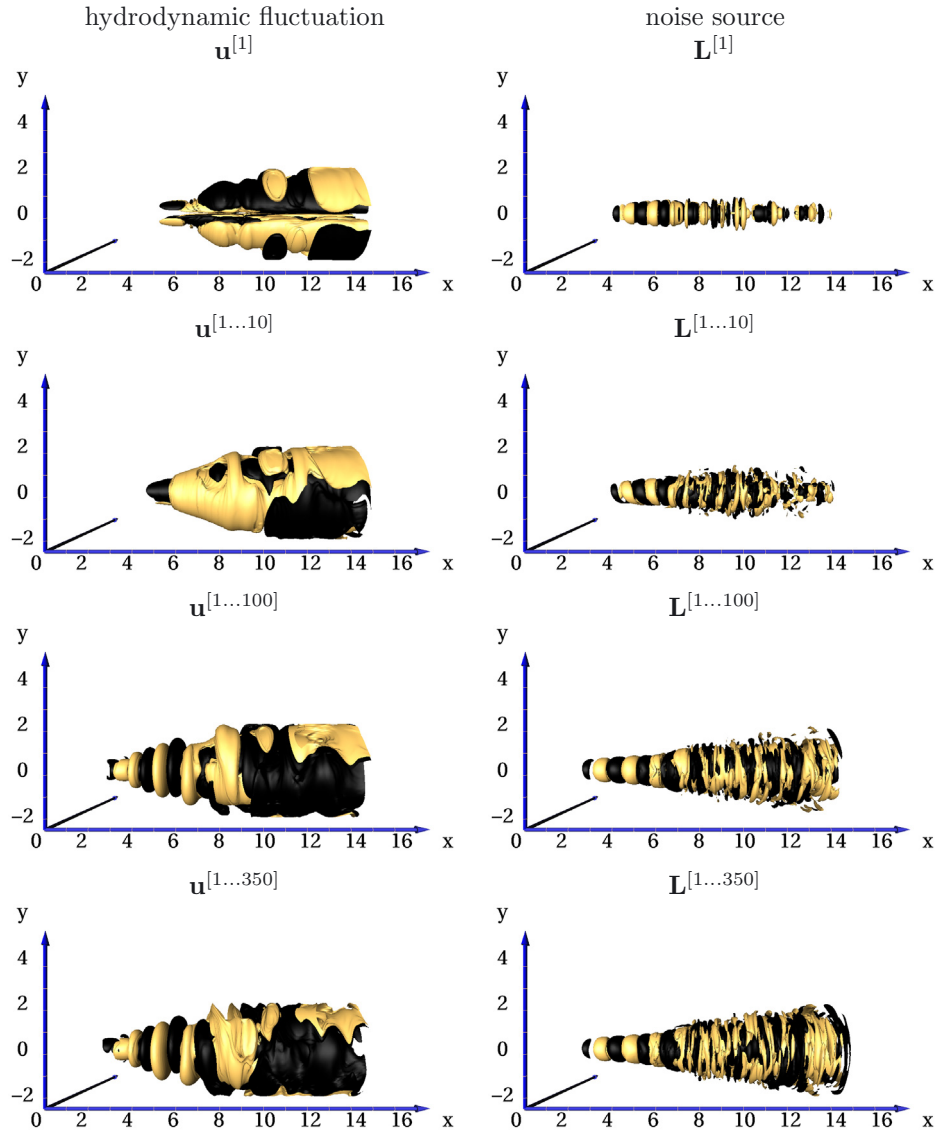


FIGURE 14. Reduced-order representations of the flow in Fig. 3. Fluctuations of the velocity field (left) and Lamb vector (right) are approximated by Galerkin expansions with N POD modes. The subfigures display reduced-order representations at $N = 1, 10, 100, 350$ top to bottom. Vector fields are visualised as in Fig. 3.

The azimuthal filtering shows that 75 percent of the turbulent kinetic energy and 70 percent of noise source fluctuations can be resolved with the first five azimuthal modes. This result highlights a difference between the compressible and incompressible jets. In the latter flow, only thirty percent of the kinetic energy is contained in the first five azimuthal modes. This difference indicates that the flow of the compressible jet is dynamically less complex. The lower complexity of compressible flow is also corroborated by more pronounced coherent vortex structures as well as pronounced nearly periodic and nearly axisymmetric pressure waves in the potential core region.

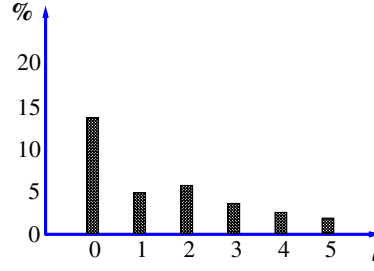


FIGURE 15. Azimuthal decomposition of the fluctuation energy $K^{(\pm l)}$ of the incompressible jet. Only 31.69% of K_Ω are resolved by the azimuthal decomposition with $l = -5, \dots, 5$.

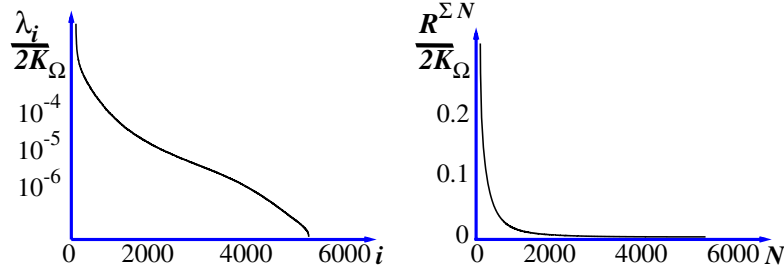


FIGURE 16. The POD spectrum of the velocity of the incompressible flow. Displayed are the POD eigenvalues λ_i (left) and the unresolved energy $R^{\Sigma N}$ (right), normalised and visualised like in Fig. 12. Here, the value R_s appearing in the definition of $R^{\Sigma N}$ is given by $R_s = 0.3169 \cdot 2K_\Omega$.

The POD of the velocity field shows distinct coherent structures in the flow field. In the initial region of the jet, axisymmetric structures are formed which disintegrate near the end of the potential core. The first 350 velocity modes resolve 50 percent of the energy, which is consistent with observations by Freund and Colonius [13]. The visualisation of the corresponding Galerkin expansion indicates that a coarse jet computation can be made by a reduced-order representation.

The same statement holds with some restriction for the noise source. Although the visualisation of the Galerkin expansion shows similar structures compared to the full source representation, only 25 percent of the noise source fluctuation is resolved by 350 velocity modes. 30 percent of the unresolved energy is contained in the higher azimuthal modes. Intriguingly, higher azimuthal modes have a larger azimuthal noise source efficiency per unit energy.

From an aeroacoustic point of view, it is left to ongoing research to what extent the resolved coherent structures contribute to the sound field. Current results of the authors indicate that the dimension of flow-field POD can be significantly further reduced if the method is tailored towards far-field noise prediction.

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