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HIGH-FREQUENCY LIMIT OF THE MAXWELL-LANDAU-LIFSHITZ EQUATIONS IN THE DIFFRACTIVE OPTICS REGIME *

LU Yong[1]

Abstract. We study the Maxwell-Landau-Lifshitz system for highly oscillating initial data, with characteristic frequencies $O(1/\varepsilon)$ and amplitude $O(1)$, over long time intervals $O(1/\varepsilon)$, in the limit $\varepsilon \to 0$. We show that a nonlinear Schrödinger equation gives a good approximation for the envelope of the solution in the time interval under consideration. This extends previous results of Colin and Lannes [1]. This text is a short version of the article [5].

Introduction

Electromagnetic wave in ferromagnetic media is described by the Maxwell-Landau-Lifshitz system

$$\text{(MLL)} \begin{cases} \partial_t E - \nabla \times H = 0, \\ \partial_t H + \nabla \times E = -\partial_t M, \\ \partial_t M \qquad\quad = -M \times H, \end{cases}$$

which admits a family of constant solutions $(E, H, M)_\alpha = (0, \alpha M_0, M_0)$ for any $\alpha > 0$ and $M_0 \in \mathbb{R}^3$. We are interested in the solutions that can be written as small, slowly variable perturbations of such constant solutions, of the form

$$\begin{cases} E(t,x) = \varepsilon \tilde{E}(\varepsilon t, \varepsilon x), \\ H(t,x) = \alpha M_0 + \varepsilon \tilde{H}(\varepsilon t, \varepsilon x), \\ M(t,x) = M_0 + \varepsilon \tilde{M}(\varepsilon t, \varepsilon x). \end{cases}$$

If the triplet (E, H, M) is a solution of (MLL), then $u = (\tilde{E}(t,x), \tilde{H}(t,x), \alpha^{\frac{1}{2}} \tilde{M}(t,x))$ satisfies the symmetric quadratic hyperbolic system

$$\partial_t u + A(\partial_x) u + \frac{1}{\varepsilon} L_0 u = B(u,u), \tag{0.1}$$

where A and L_0 are defined by

$$A(\partial_x) := \begin{pmatrix} 0 & -\partial_x \times & 0 \\ \partial_x \times & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad L_0 := \begin{pmatrix} 0 & 0 & 0 \\ 0 & -M_0 \times & \alpha^{\frac{1}{2}} M_0 \times \\ 0 & \alpha^{\frac{1}{2}} M_0 \times & -\alpha M_0 \times \end{pmatrix}, \tag{0.2}$$

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[1] Institut de Mathématiques de Jussieu, Université Paris Diderot. e-mail: luyong@math.jussieu.fr

and the bilinear form $B(\cdot,\cdot)$ is defined for any $u=(u^1,u^2,u^3), v=(v^1,v^2,v^3)\in\mathbb{R}^3\times\mathbb{R}^3\times\mathbb{R}^3$, by

$$B(u,v):=\frac{1}{2}\begin{pmatrix}0\\ \alpha^{-\frac{1}{2}}(u^3\times v^2+v^3\times u^2)\\ -(u^3\times v^2+v^3\times u^2)\end{pmatrix}. \tag{0.3}$$

Without loss of generality, we take $\alpha=1$ and $M_0=(1,0,0)$.

For some technique reasons about localizing the resonances, we restrict our problem to one spatial dimension, where the equation (0.1) becomes

$$\partial_t v+A(e_1)\partial_y v+\frac{L_0 v}{\varepsilon}=B(v,v), \tag{0.4}$$

where $y\in\mathbb{R}^1$ is the spatial variable, and $e_1=(1,0,0)\in\mathbb{R}^3$. We consider highly oscillatory initial data

$$v(0,y)=v_0(y):=a(y)e^{iky/\varepsilon}+\overline{a(y)}e^{-iky/\varepsilon}+\varepsilon a_1(y,ky/\varepsilon)+\varepsilon^2 a_2(y,ky/\varepsilon). \tag{0.5}$$

Our *goal* is to justify the cubic Schrödinger approximation over long time interval of order $O(1/\varepsilon)$.

For all $(\lambda,\xi)\in\mathbb{R}^{1+1}$, we introduce the matrices $L(i(\lambda,\xi))=i\lambda+A(e_1)i\xi+L_0$ and the characteristic variety $\mathrm{Char}L(i(\lambda,\xi))=\{(\lambda,\xi)|\det\ L(i(\lambda,\xi))=0\}$. For $\xi\in\mathbb{R}^1$, we have spectral decomposition $A(e_1)\xi+L_0/i=\sum_{j=1}^9\lambda_j(\xi)\Pi_j(\xi)$. We denote the total projections, $\Pi_0(\xi)=\sum_{j=1}^6\Pi_j(\xi)$, $\Pi_s(\xi)=\sum_{j=7}^9\Pi_j(\xi)$. By direct calculation, we have that for any $\xi\in\mathbb{R}$,

$$\Pi_0(\xi)=\mathrm{diag}\{0,1,1,0,1,1,0,1,1\},\ \ \Pi_s(\xi)=\mathrm{diag}\{1,0,0,1,0,0,1,0,0\}. \tag{0.6}$$

1. The main difficulties

The equation (0.4) is symmetric hyperbolic. For fixed $\varepsilon>0$, the local existence, uniqueness and regularity of solutions to (0.4)-(0.5) in Sobolev space are classical. Our goal here is to describe solutions in the high-frequency limit $\varepsilon\to 0$ over long time $O(1/\varepsilon)$. As $\varepsilon\to 0$, there are three main issues.

First, because of the fast oscillations, the Sobolev norm is unbounded when $\varepsilon\to 0$: for any $s>0$,

$$|v(0)|_{H^s}=|a(y)e^{iky/\varepsilon}+...|_{H^s}=O(\varepsilon^{-s})\to\infty.$$

By the classical theory of symmetric hyperbolic systems, the existence time converges to 0 as $\varepsilon\to 0$. We go around this issue by introducing *profiles* as in [2](see Section 2.1).

Second, the classical H^s energy estimate for semilinear symmetric hyperbolic operators yields

$$|v(t)|_{H^s}\le|v(0)|_{H^s}+C\int_0^t|v(t')|_{H^s}dt',$$

where the constant C depends on $|v|_{L^\infty}$, implying, by Gronwall's lemma, the bound

$$|v(t)|_{H^s}\le|v(0)|_{H^s}e^{Ct}.$$

In long time of size $O(1/\varepsilon)$, the upper bound diverges to infinity as $\varepsilon\to 0$. This issue is dealt with by normal form reductions, see section 3.2.

Third, we are confronted with the issue of deriving uniform bounds for the corrector terms of the ansatz. If we denote $V=V_0+\varepsilon V_1+...$ the WKB expansion, we want indeed to guarantee that εV_1 is $o(1)$ under the whole interval under consideration, otherwise the significance of the WKB expansion as an asymptotic expansion breaks down. However there typically arises an equation of the form

$$(\partial_t + \rho\partial_y)a = b, \tag{1.1}$$

where a is a component of V_1 and b depends on the leading term V_0. For the trivial datum $a(0,y) = 0$, the solution is $a(t,y) = \int_0^t b(t-s, y-\rho s)ds$, giving typically the estimate $\|a(t)\|_{H^s} \leq t\|b\|_{L^\infty(H^s)}$ and nothing better. In the long time interval $[0, T/\varepsilon]$, the upper bound is of order $O(1/\varepsilon)$ implying that εV_1 could well be $O(1)$. This phenomenon is called *secular growth* and was extensively studied by Lannes in [3]. We eliminate this secular growth here by choosing prepared initial data (See Section 3.2, in particular (3.6)) such that we could have the source term b to be null, then no more secular growth for a in (1.1).

2. Description of the results

In the context of Maxwell-Landau-Lifshitz system, we improve the analysis of Colin and Lannes [1] from time $O(|\ln\varepsilon|)$ up to $O(1/\varepsilon)$, assuming that the data (leading term and corrector) are prepared. For general data which only satisfy a polarization condition for the leading term, we show the stability over times $O(\varepsilon^{-\alpha}|\ln\varepsilon|)$ for any $0 < \alpha < 1$.

2.1. **Change of variable and profile**

To solve the Cauchy problem (0.4)-(0.5) up to time $O(1/\varepsilon)$, we consider a representation of the solutions by profiles depending periodically on the fast variable: $v(t,y) = V(\tau,t,y,\theta)|_{\tau=\frac{t}{\varepsilon},\theta=\frac{ky-\omega t}{\varepsilon}}$. We remark that, if a smooth function $V(\tau,t,y,\theta)$ solves the Cauchy problem

$$\begin{cases} \partial_\tau V + \dfrac{1}{\varepsilon}\{\partial_t + A(e_1)\partial_y\}V + \dfrac{1}{\varepsilon^2}\{-\omega\partial_\theta + A(e_1)k\partial_\theta + L_0\}V = \dfrac{1}{\varepsilon}B(V,V), \\ V(0,0,y,\theta) = V_0(y,\theta) := a(y)e^{i\theta} + \overline{a(y)}e^{-i\theta} + \varepsilon a_1(y,\theta) + \varepsilon^2 a_2(y,\theta) \end{cases} \tag{2.1}$$

on the time interval $[0,T]_\tau \times [0,T/\varepsilon]_t$, then the function defined as $v(t,y) = V(\varepsilon t, t, y, \frac{ky-\omega t}{\varepsilon})$ solves the Cauchy problem (0.4)-(0.5) on the time interval $[0, T/\varepsilon]$.

We take $(\omega, k) \in \mathrm{Char}L(i(\lambda,\xi))$ which means that we choose a time frequency that is compatible with the initial space frequency. We assume that the leading term of the initial data satisfies the polarization condition: $a(y,\theta) \in \ker L(i(\omega,k))$. We also assume $a(y) \in H^s$, $a_1(y,\theta) \in H^2(\mathbb{T}_\theta, H^{s-1}(\mathbb{R}^1_y))$ and $a_2(y,\theta) \in H^2(\mathbb{T}_\theta, H^{s-2}(\mathbb{R}^1_y))$, where $s > 9 + 1/2$.

2.2. **Approximate profile and approximate solution**

We construct an approximate profile to (2.1) by WKB expansion:

Proposition 2.1. *There exists $V^a \in L^\infty([0,T]_\tau \times [0,\frac{T}{\varepsilon}]_t, H^1(\mathbb{T}_\theta, H^{s-2}(\mathbb{R}^1_y)))$ for some positive $T > 0$ independent of ε, such that V^a solves the following Cauchy problem on $[0,T]_\tau \times [0,\frac{T}{\varepsilon}]_t$:*

$$\begin{cases} \partial_\tau V^a + \dfrac{1}{\varepsilon}\{\partial_t + A(e_1)\partial_y\}V^a + \dfrac{1}{\varepsilon^2}\{-\omega\partial_\theta + A(e_1)k\partial_\theta + L_0\}V^a = \dfrac{1}{\varepsilon}B(V^a,V^a) + \varepsilon R, \\ V^a(0,0,y,\theta) = V_0(y,\theta) + \varepsilon b(y,\theta) + \varepsilon^2 b_1(y,\theta), \end{cases} \tag{2.2}$$

for some $R(\tau,t,y,\theta) \in L^\infty([0,T]_\tau \times [0,\frac{T}{\varepsilon}]_t, H^1(\mathbb{T}_\theta, H^{s-2}(\mathbb{R}^1_y)))$, $(b(y,\theta), b_1(y,\theta)) \in H^1(\mathbb{T}_\theta, H^{s-2}(\mathbb{R}^1_y))$.

Then if we define $v^a := V^a(\varepsilon t, t, y, \frac{ky-\omega t}{\varepsilon})$, $v^a \in L^\infty([0,\frac{T}{\varepsilon}]_t \times \mathbb{R}^1_y)$ satisfies the following Cauchy problem over the time interval $[0,\frac{T}{\varepsilon}]_t$:

$$\begin{cases} (\partial_t + A(e_1)\partial_y)v^a + \dfrac{1}{\varepsilon}L_0 v^a = B(v^a, v^a) + \varepsilon R, \\ v^a(0,0,y) = v_0(y) + \varepsilon b(y, \dfrac{ky}{\varepsilon}) + \varepsilon^2 b_1(y, \dfrac{ky}{\varepsilon}). \end{cases} \tag{2.3}$$

2.3. **Error estimate**

We show that v^a converges to the exact solution as $\varepsilon \to 0$ over its existence time.

Theorem 2.2. *There exists a constant $T^* > 0$ and a small constant $\varepsilon_0 > 0$, such that for all $T < T^*$ and $0 < \varepsilon < \varepsilon_0$:*

i. *Over the time interval $[0, T/\varepsilon]$, the Cauchy problem (0.4)-(0.5) admits a unique solution v of the form $v(t,y) = V(\varepsilon t, y, \frac{ky-\omega t}{\varepsilon})$, with $V(\tau, y, \theta) \in L^\infty([0,T]_\tau, H^1(\mathbb{T}_\theta, H^{s-2}(\mathbb{R}^1_y)))$.*

ii. *We have the following error estimates*

$$\|\Pi_0(v - v^a)\|_{L^\infty([0,\frac{T}{\varepsilon}]\times\mathbb{R}^1)} \le C_s(\varepsilon + T), \quad \|\Pi_s(v - v^a)\|_{L^\infty([0,\frac{T}{\varepsilon}]\times\mathbb{R}^1)} \le C_s\varepsilon. \tag{2.4}$$

For smaller $T = \varepsilon^{(1-\alpha)}|\ln\varepsilon|$ for any $0 < \alpha < 1$, we have

$$\|(v - v^a)\|_{L^\infty([0,\varepsilon^{-\alpha}|\ln\varepsilon|]\times\mathbb{R}^1)} \le C_s\varepsilon^{(1-\alpha)}|\ln\varepsilon|. \tag{2.5}$$

iii. *Additionally assuming that a_1 is given by (3.6), we have the better error estimates*

$$\|\Pi_0(v - v^a)\|_{L^\infty([0,\frac{T}{\varepsilon}]\times\mathbb{R}^1)} \le C_s\varepsilon, \quad \|\Pi_s(v - v^a)\|_{L^\infty([0,\frac{T}{\varepsilon}]\times\mathbb{R}^1)} \le C_s\varepsilon^2. \tag{2.6}$$

The constant C_s is independent of ε and v, and depends on the sum of norms $\|a\|_{H^s} + \|a_1\|_{H^2(\mathbb{T}_\theta, H^{s-1})} + \|a_2\|_{H^2(\mathbb{T}_\theta, H^{s-2})}$.

3. Idea of the proof

3.1. **WKB expansion**

The first step is done by a standard three-scale WKB expansion. An approximate profile V^a is constructed as the solution of

$$\begin{cases} \partial_\tau V^a + \dfrac{1}{\varepsilon}\{\partial_t + A(e_1)\}\partial_y V^a + \dfrac{1}{\varepsilon^2}\{-\omega\partial_\theta + A(e_1)k\partial_\theta + L_0\}V^a = \dfrac{1}{\varepsilon}B(V^a, V^a) + \varepsilon R, \\ V^a(0,0,y,\theta) = V_0(y,\theta) + \varepsilon b(y,\theta) + \varepsilon^2 b_1(y,\theta), \end{cases}$$

for some uniformly bounded functions (R, b, b_1) over $[0,T]_\tau \times [0, \frac{T}{\varepsilon}]_t$. The profile V^a has the form

$$V^a = (V_{01}e^{i\theta} + \overline{V}_{01}e^{-i\theta}) + \varepsilon(V_{10} + ...) + \varepsilon^2(...), \tag{3.1}$$

where the components of V_{01} satisfy cubic Schrödinger equations. This proves Proposition 2.1.

3.2. **Normal form reduction and error estimate**

We now look for the exact solution of (0.4)-(0.5) as a perturbation of $v^a = V^a(\varepsilon t, t, y, (ky - \omega t)/\varepsilon)$ in the form

$$v(t,y) = v^a(t,y) + \varepsilon W(\varepsilon t, y, \frac{ky - \omega t}{\varepsilon}).$$

Then the equation in W is

$$\begin{cases} \partial_\tau W + \dfrac{i}{\varepsilon^2}\mathcal{A}W + \dfrac{\omega\partial_\theta}{\varepsilon^2}W = \dfrac{2}{\varepsilon}B(V^a, W) - R, \\ W(0,y,\theta) = -b(y,\theta) - \varepsilon b_1(y,\theta). \end{cases} \tag{3.2}$$

The operator $\mathcal{A}$ has the form $\mathcal{A} = A(e_1)(\varepsilon D_y + kD_\theta) + L_0/i$ and D is defined as $D = \partial/i$.

We introduce $W_1 = (\Pi_0 W, \Pi_s W)^t$, so that

$$\partial_\tau W_1 + \frac{i}{\varepsilon^2}\mathcal{A}_1 W_1 - \frac{1}{\varepsilon^2}\omega\partial_\theta W_1 = \frac{2}{\varepsilon}B_1(V^a)W_1 + E_1(W_1, W_1) + R_1. \tag{3.3}$$

The singular source term is

$$B_1(V^a) = \begin{pmatrix} \Pi_0 B(V^a)\Pi_0 & \Pi_0 B(V^a)\Pi_s \\ \Pi_s B(V^a)\Pi_0 & \Pi_s B(V^a)\Pi_s \end{pmatrix} = \begin{pmatrix} O(\varepsilon) & O(1) \\ B_{s0} & O(\varepsilon) \end{pmatrix}.$$

Because constructive interaction of waves between low- and high-frequency do occur for the MLL system, the $O(1)$ term in the top right block of the above interaction matrix can be interpreted as an absence of transparency [2].

We now introduce the linear change of variable (normal form reduction) in order to transform the $O(1)$ bottom left block B_{s0} in the singular source into $O(\varepsilon)$ term:

$$W_2 := (\mathrm{Id} + \varepsilon N)^{-1} W_1 = (\mathrm{Id} - \varepsilon N)W_1, \;\; N := \begin{pmatrix} 0 & 0 \\ N_{s0} & 0 \end{pmatrix}.$$

Then the equation in W_2 is of the form

$$\begin{aligned} \partial_\tau W_2 + \frac{i}{\varepsilon^2}\mathcal{A}W_2 - \frac{1}{\varepsilon^2}\omega\partial_\theta W_2 \;\; &= \;\; \frac{1}{\varepsilon}\{-i[\mathcal{A}, N] + \omega\partial_\theta N + 2\begin{pmatrix} 0 & 0 \\ B_{s0} & 0 \end{pmatrix} + \varepsilon B_1\}W_2 \\ &+ \frac{2}{\varepsilon}\begin{pmatrix} 0 & O(1) \\ 0 & 0 \end{pmatrix} W_2 + \begin{pmatrix} 2\Pi_0 B(W_{20}, W_{2s}) \\ \Pi_s B(W_{20}, W_{20}) \end{pmatrix} + \begin{pmatrix} \Pi_0 R \\ \Pi_s R \end{pmatrix} + O(\varepsilon). \end{aligned} \tag{3.4}$$

We look for N as a solution of the homological equation

$$-i[\mathcal{A}, N] + \omega\partial_\theta N + 2\begin{pmatrix} 0 & 0 \\ B_{s0} & 0 \end{pmatrix} + \varepsilon B_1 = O(\varepsilon^2). \tag{3.5}$$

By projecting onto characteristic modes, we can see that equation (3.5) reduces to a family of equations $\Phi_{pjj'}\Pi_j N_{s0}\Pi_{j'} = \Pi_j B_{s0}\Pi_{j'}$, where Π_j are the characteristic projectors and the $\Phi_{pjj'} = \lambda_{j'} - \lambda_j - p\omega$ are the *resonance* relations, see [4].

Thus, the resolution of (3.5) requires the projection of B_{s0} to be small where $\Phi_{pjj'}$ is small. This condition on the source term is called *transparency* by Joly, Métivier and Rauch [2]. We show by direct computation that B_{s0} is transparent. Then the equation in W_2 becomes

$$\begin{cases} \partial_\tau W_2 + \dfrac{i}{\varepsilon^2}\mathcal{A}W_2 - \dfrac{1}{\varepsilon^2}\omega\partial_\theta W_2 = \begin{pmatrix} O(1) & O(\varepsilon^{-1}) \\ O(\varepsilon) & 0 \end{pmatrix}\begin{pmatrix} W_{20} \\ W_{2s} \end{pmatrix} + \begin{pmatrix} 2\Pi_0 B(W_{20}, W_{2s}) \\ \Pi_s B(W_{20}, W_{20}) \end{pmatrix} + O(\varepsilon), \\ W_2(0,y,\theta) = -\begin{pmatrix} \Pi_0 b \\ \Pi_s b \end{pmatrix} + O(\varepsilon). \end{cases}$$

We then rescale the solution $W_3 = \begin{pmatrix} W_{30} \\ W_{3s} \end{pmatrix} := \begin{pmatrix} \varepsilon W_{20} \\ W_{2s} \end{pmatrix}$. The system in W_3 has the form

$$\begin{cases} \partial_\tau W_3 + \dfrac{i}{\varepsilon^2}\mathcal{A}_1 W_3 - \dfrac{1}{\varepsilon^2}\omega\partial_\theta W_3 = \begin{pmatrix} O(1) & O(1) \\ O(1) & 0 \end{pmatrix}\begin{pmatrix} W_{30} \\ W_{3s} \end{pmatrix} + \begin{pmatrix} 2\Pi_0 B(W_{30}, W_{3s}) \\ \varepsilon^{-2}\Pi_s B(W_{30}, W_{30}) \end{pmatrix} + O(\varepsilon), \\ W_3(0, y, \theta) = -\begin{pmatrix} \varepsilon\Pi_0 b \\ \Pi_s b \end{pmatrix} + O(\varepsilon). \end{cases}$$

We show by direct calculation that the singular bilinear term $\varepsilon^{-2}\Pi_s B(W_{30}, W_{30})$ satisfies a *strong transparency* condition, meaning that it can be eliminated by a *nonlinear* change of variable such as introduced by Joly, Métivier and Rauch [2]. This implies uniform estimates for W_3 over the time interval under consideration. Back to W_2, we only have that $W_{20} = \varepsilon^{-1}W_{30} = O(\varepsilon^{-1})$. It follows $|\Pi_0(v - v^a)| = O(1)$. This is the first error estimate in Theorem 2.2.

This result can be improved to show that v^a does approximate the exact solution. We consider the additional preparation condition for the initial corrector a_1:

$$a_1(y, \theta) = V_{10}(0, 0, y) = \frac{4k\delta}{\omega\rho}(1 - \frac{k\rho}{\omega})\begin{pmatrix} 0 \\ -e_1 \\ e_1 \end{pmatrix}|a_0(y)|^2 + \tilde{a}(y, \theta), \tag{3.6}$$

where $\widetilde{a}$ satisfies $\Pi_s\widetilde{a} = 0$, ρ is the group velocity (in function of ω and k) and $\delta = \pm 1$ depends on (ω, k). Under this condition, we have that $\Pi_s(V - V_a)(0, 0, y, \theta) = O(\varepsilon^2)$. Then we rescale the solution by defining $W_4 = \begin{pmatrix} W_{40} \\ W_{4s} \end{pmatrix} := \begin{pmatrix} W_{20} \\ \varepsilon^{-1}W_{2s} \end{pmatrix}$. Similar arguments show that W_4 is uniformly bounded over the time interval under consideration. Then we obtain the estimates:

$$|\Pi_0(V - V^a)| = O(\varepsilon), \quad |\Pi_s(V - V^a)| = O(\varepsilon^2).$$

This means that the WKB approximate profile stays close to the exact profile over its existence time with an error estimate that is comparable to the initial error. This gives (2.6) in Theorem 2.2.

References

[1] T. Colin, D. Lannes, *Justification of and long-wave correction to Davey-Stewartson systems from quadratic hyperbolic systems*, Discr. and Contin. Dynam. Sys., vol. 11, n. 1 (2004), 83-100

[2] J.-L. Joly, G. Métivier, J. Rauch, *Transparent nonlinear geometric optics and Maxwell-Bloch equations*, J. Diff. Eq., vol. 166 (2000), 175-250.

[3] D. Lannes, *Secular growth estimates for hyperbolic systems*, J. Diff. Eq.,vol. 190 (2003), 466–503.

[4] B. Texier, *Derivation of the Zakharov equations*, Arch. Rational Mech. Anal 184 (2007), 121–183.

[5] LU Yong, *High-frequency limit of the Maxwell-Landau-Lifshitz equations in the diffractive optics regime*, preprint (2012).