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TWO-AIRCRAFT OPTIMAL CONTROL PROBLEM. THE IN-FLIGHT NOISE REDUCTION *, **

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Abstract. The aim of this paper is to present and solve a mathematical model of a two-aircraft optimal control problem reducing the noise on the ground during the approach. The mathematical modelization of this problem is a non-convex optimal control governed by ordinary non-linear differential equations. To solve this problem, A direct method and a Runge-Kutta RK4 discretization schema are used. This discretization schema is chosed because it is a sufficiently high order and it does-not require computation of the partial derivatives of the aircraft dynamic. The Nonlinear Interior point Trust Region Optimization solver KNITRO is applied. A large set of numerical experiments is presented. The obtained results give feasible trajectories with a significant noise reduction.

Introduction

In this work, a theoretical model of noise optimization is developped while maintaining a reliable evolution of the flight procedures of two commercial aircraft on approach. In particular, this work is focused on aircraft coupling noise levels and energetic consumption. These two-aircraft are supposed to land successively on one runway without conflict [20].

The model considered here is non-convex and non-linear optimal control problem leading to a system of non-linear ordinary differential equations [6]. The aircraft dynamic is described by a three dimensional set of non-linear ordinary differential equations subjected to state and control constraints. The functional to be minimized describes the overall levels of noise collected on the ground, emitted by the two mentioned aircraft. The formulation of the problem takes into account several kinds of constraints such as aircraft stability, performance and flight safety. The Nonlinear Interior point Trust Region Optimization solver 'KNITRO' [21] is used to solve the obtained algebric non-linear system of equation implemented by A Modeling Language for Mathematical Programming 'AMPL' [9, 17].

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The two-aircraft flight dynamic, the noise levels, the constraints, the mathematical basic equations of the two-aircraft acoustic optimal control problem and the discretization scheme are presented in sections 2 and 3 while the numerical experiments are presented in the last section.

1. Mathematical description of the basic equations

1.1. **Aircraft dynamic equations**

The equations of 3D-motion of each aircraft $A_i, i \in \{1,2\}$ read [3,4]:

$$\begin{cases} \dot{m}_i &= -2.01\times 10^{-5}\frac{(\Phi-\mu_i-\frac{K_i}{\eta_c})\sqrt{\Theta}}{\sqrt{5\eta_n(1+\eta_{tf_i}\lambda)}\sqrt{G_i+0.2M_i^2\frac{\eta_{d_i}}{\eta_{tf_i}}\lambda-(1-\lambda)M_i}}P_0\delta_{xi}\frac{\rho_i}{\rho_0}(1-M_i+\frac{M_i^2}{2}),\\ \dot{V_{a_i}} &= \frac{1}{m_i}[-m_i g sin\gamma_{a_i} - \frac{1}{2}\rho_i S V_{a_i}^2 C_{D_i} + (cos\alpha_{a_i}cos\beta_{a_i} + sin\beta_{a_i} + sin\alpha_{a_i}cos\beta_{a_i})F_{x_i}\\ &+C_{SR_i}P_0\delta_{xi}\frac{\rho_i}{\rho_0}(1-M_i+\frac{M_i^2}{2})u_i - m_i\Delta A_u^i],\\ \dot{\beta_{a_i}} &= \frac{1}{m_i V_{a_i}}[m_i g cos\gamma_{a_i} sin\mu_{a_i} + \frac{1}{2}\rho_i S V_{a_i}^2 C_{y_i} + [-cos\alpha_{a_i}sin\beta_{a_i} + cos\beta_{a_i} - sin\alpha_{a_i}sin\beta_{a_i}]F_{y_i},\\ &+C_{SR_i}P_0\delta_{xi}\frac{\rho_i}{\rho_0}(1-M_i+\frac{M_i^2}{2})v_i - m_i\Delta A_v^i],\\ \dot{\alpha_{a_i}} &= \frac{1}{m_i V_{a_i}cos\beta_{a_i}}[m_i g cos\gamma_{a_i}cos\mu_{a_i} - \frac{1}{2}\rho_i S V_{a_i}^2 C_{L_i} + [-sin\alpha_{a_i} + cos\alpha_{a_i}]F_{z_i}\\ &+C_{SR_i}P_0\delta_{xi}\frac{\rho_i}{\rho_0}(1-M_i+\frac{M_i^2}{2})w_i - m_i\Delta A_w^i],\\ \dot{p}_i &= \frac{C}{AC-E^2}\{r_i q_i(B-C) - E p_i q_i + \frac{1}{2}\rho_i S l V_{a_i}^2 C_{l_i} + \sum_{j=1}^2 F_j[y_{Mij}^b cos\beta_{mij}sin\alpha_{mij} - z_{Mij}^b sin\beta_{mij}]\},\\ &+\frac{E}{AC-E^2}\{p_i q_i(A-B) - E r_i q_i + \frac{1}{2}\rho_i S l V_{a_i}^2 C_{ni} + \sum_{j=1}^2 F_j[x_{Mij}^b sin\beta_{mij} - y_{Mij}^b cos\beta_{mij}cos\alpha_{mij}]\},\\ \dot{q}_i &= \frac{1}{B}\{-r_i p_i(A-C) - E(p_i^2 - r_i^2) + \frac{1}{2}\rho_i S l V_{a_i}^2 C_{mi}\\ &+\sum_{j=1}^2 F_j[z_{Mij}^b cos\beta_{mij}cos\alpha_{mij} - x_{Mij}^b cos\beta_{mij}sin\alpha_{mij}]\},\\ \dot{r}_i &= \frac{E}{AC-E^2}\{r_i q_i(B-C) + E p_i q_i + \frac{1}{2}\rho_i S l V_{a_i}^2 C_{l_i} + \sum_{j=1}^2 F_j[y_{Mij}^b cos\beta_{mij}sin\alpha_{mij} - z_{Mij}^b sin\beta_{mij}]\},\\ &+\frac{A}{AC-E^2}\{p_i q_i(A-B) - E r_i q_i + \frac{1}{2}\rho_i S l V_{a_i}^2 C_{ni} + \sum_{j=1}^2 F_j[x_{Mij}^b sin\beta_{mij} - y_{Mij}^b cos\beta_{mij}cos\alpha_{mij}]\},\\ \dot{X}_{G_i} &= V_{a_i}cos\gamma_{a_i}cos\chi_{a_i} + u_w, \dot{Y}_{G_i} = V_{a_i}cos\gamma_{a_i}sin\chi_{a_i} + v_w, \dot{Z}_{G_i} = -V_{a_i}sin\gamma_{a_i} + w_w,\\ \dot{\phi}_i &= p_i + q_i sin\phi_i tan\theta_i + r_i cos\phi_i tan\theta_i, \dot{\theta}_i = q_i cos\phi_i - r_i sin\phi_i, \dot{\psi}_i = \frac{sin\phi_i}{cos\theta_i}q_i + \frac{cos\phi_i}{cos\theta_i}r_i, \end{cases} \tag{1}$$

where $j \in \{1,2\}$ stands for the first and second engine of each aircraft i, the expressions $A = I_{xx}, B = I_{yy}, C = I_{zz}, E = I_{xz}$ are the inertia moments of the aircraft, ρ_i is the air density, S is the aircraft reference area, l is the aircraft reference length, g is the acceleration due to gravity, $C_{D_i} = C_{D0} + kC_{Li}^2$ is the drag coefficient, $C_{yi} = C_{y\beta}\beta + C_{yp}\frac{pl}{V} + C_{yr}\frac{rl}{V} + C_{Y\delta_l}\delta_{li} + C_{Y\delta_n}\delta_{ni}$ is the lateral forces coefficient, $C_{Li} = C_{L\alpha}(\alpha_a - \alpha_{a0}) + C_{L\delta_m}\delta_{mi} + C_{LM}M_i + C_{Lq}\frac{q_a^b l}{V}$ is the lift coefficient, $C_{li} = C_{l\beta}\beta + C_{lp}\frac{pl}{V} + C_{lr}\frac{rl}{V} + C_{l\delta_l}\delta_{li} + C_{l\delta_n}\delta_{ni}$ is the rolling moment coefficient, $C_{mi} = C_{m0} + C_{m\alpha}(\alpha - \alpha_0) + C_{m\delta_m}\delta_{mi}$ is the pitching moment coefficient, $C_{ni} = C_{n\beta}\beta + C_{np}\frac{pl}{V} + C_{nr}\frac{rl}{V} + C_{n\delta_l}\delta_{li} + C_{n\delta_n}\delta_{ni}$ is the yawing moment coefficient, $(x_{Mij}^b, x_{Mij}^b, x_{Mij}^b)$ is the position of the engine in the body frame, P_0 is the full thrust, ρ_0 is the atmospheric density at the ground, $F = (F_{xi}, F_{yi}, F_{zi})$ is the propulsive force, $V_{ai} = (u_i, v_i, w_i)$ is the aerodynamic speed, $(\Delta A_u^i, \Delta A_v^i, \Delta A_w^i)$ is the complementary acceleration, (u_w, v_w, w_w) is the wind velocity, β_{mij} is the yaw setting of the engine and α_{mij} is the pitch setting of the engine. The mass change is reflected in the aircraft fuel consumption as described by E. Torenbeek [19] where the specific consumption is $C_{SR_i} = 2.01\times 10^{-5}\frac{(\Phi-\mu_i-\frac{K_i}{\eta_c})\sqrt{\Theta}}{\sqrt{5\eta_n(1+\eta_{tf_i}\lambda)}\sqrt{G_i+0.2M_i^2\frac{\eta_{d_i}}{\eta_{tf_i}}\lambda-(1-\lambda)M_i}}$ with the generating function $G_i = (\Phi - \frac{K_i}{\eta_c})(1 - \frac{1.01}{\eta_i^{\frac{\nu-1}{\nu}}(K_i+\mu_i)(1-\frac{K_i}{\Phi\eta_c\eta_t})}), K_i = \mu_i(\epsilon_c^{\frac{\nu-1}{\nu}} - 1), \mu_i = 1 + \frac{\nu-1}{2}M_i^2$. The nomenclature of engine performance variables are given by G_i the gas generator power function, G0 the gas generator power function (static, sea level), K the temperature function of compression process, M_i the flight Mach number, T4 the turbine Entry total Temperature, T0 the ambient temperature at sea level, T the flight temperature, while the nomenclature of engines yields is $\eta_c = 0.85$ the isentropic compressor efficiency, $\eta_{d_i} = 1 - 1.3(\frac{0.05}{Re^{\frac{1}{5}}})^2(\frac{0.5}{M_i})^2\frac{L}{D}$, the isentropic fan intake duct efficiency, L the duct length, D the inlet diameter, Re the Reynolds number at the entrance of the nozzle, $\eta_{f_i} = 0.86 - 3.13 \times 10^{-2} M_i$ the isentropic fan efficiency, $\eta_i = \frac{1+\eta_{d_i}\frac{\gamma-1}{2}M_i^2}{1+\frac{\gamma-1}{2}M_i^2}$ the gas generator intake stagnation pressure ratio, $\eta_n = 0.97$ the isentropic efficiency of expansion process in nozzle, $\eta_t = 0.88$ the isentropic turbine efficiency $\eta_{tf_i} = \eta_t \eta_{f_i}$, ϵ_c the overall pressure ratio (compressor), ν the ratio of specific heats $\nu = 1.4$, λ the bypass ratio, μ_i the ratio of stagnation to static temperature of ambient air, Φ the nondimensional turbine entry temperature $\Phi = \frac{T_4}{T}$ and Θ the relative ambient temperature $\Theta = \frac{T}{T_0}$. The expressions $\alpha_{ai}(t), \beta_{ai}(t), \theta_i(t), \psi_i(t), \phi_i(t), V_{a_i}(t), X_{G_i}(t), Y_{G_i}(t), Z_{G_i}(t), p_i(t), q_i(t), r_i(t), m_i(t)$ are respectively the attack angle, the aerodynamic sideslip angle, the inclination angle, the cup, the roll angle, the airspeed, the position vectors, the roll velocity of the aircraft relative to the earth, the pitch velocity of the aircraft relative to the earth, the yaw velocity of the aircraft relative to the earth and the aircraft mass. The system (2) could be written in a simplified form

$$\begin{array}{l} \frac{d\mathbf{y}_i(t)}{dt} = \mathbf{f}_i(\mathbf{y}_i(t), \mathbf{u}_i(t)), \\ \mathbf{y}_i(t) = (\alpha_{ai}(t), \beta_{ai}(t), \theta_{ai}(t), \psi_{ai}(t), \phi_i(t), V_{a_i}(t), X_{G_i}(t), Y_{G_i}(t), Z_{G_i}(t), p_i(t), q_i(t), r_i(t), m_i(t)) \\ \mathbf{u}_i(t) = (\delta_{l_i}(t), \delta_{m_i(t)}, \delta_{n_i}(t), \delta_{x_i}(t)) \end{array} \tag{2}$$

henceforth $\mathbf{y}_i$ is called a state function and the expressions $\delta_{l_i}(t), \delta_{m_i(t)}, \delta_{n_i}(t), \delta_{x_i}(t)$ are respectively the roll control, the pitch control, the yaw control and the thrust control. The dynamics relationship can be written as: $\dot{\mathbf{y}}_i(t) = \mathbf{f}_i(\mathbf{y}_i, \mathbf{u}_i, t), \forall t \in [0, T], y_i(0) = y_{i0}$.

1.2. **The objective function model**

Let us define the quantity named the Sound Exposure Level 'SEL' [1,10,18]: $SEL = 10log\left[\int_{t'} 10^{0.1 L_{A1,dt}(t)} dt\right]$ where t' is the noise event interval. $[t_{10}, t_{1f}]$ and $[t_{20}, t_{2f}]$ are the respective approach intervals for the first and second aircraft, the objective function is calculated as:

$$\begin{array}{rl} SEL_G & = 10\log\{\frac{1}{t_{2f}-t_{10}}[(t_{20}-t_{10})\int_{t_{10}}^{t_{20}} 10^{0.1L_{A1}(t)}dt + (t_{1f}-t_{20})\int_{t_{20}}^{t_{1f}} 10^{0.1L_{A1}(t)}dt \\ & +(t_{1f}-t_{20})\int_{t_{20}}^{t_{1f}} 10^{0.1L_{A2}(t)}dt + (t_{2f}-t_{1f})\int_{t_{1f}}^{t_{2f}} 10^{0.1L_{A2}(t)}dt,]\}, t \in [t_{10}, t_{2f}] \end{array} \tag{3}$$

where the cost function SEL_G is the cumulated two-aircraft noise. Expressions $L_{A1}(t), L_{A2}(t)$ are equivalent and reflect the aircraft jet noise given by the formula [1,13]: $L_{A1}(t) = 141 + 10\log\left(\frac{\rho_1}{\rho_i}\right)^w + 10\log\left(\frac{V_e}{c}\right)^{7.5} + 10\log s_1 + 3\log\left(\frac{2s_1}{\pi d_1^2} + 0.5\right) + 5\log\frac{\tau_1}{\tau_2} + 10\log\left[\left(1-\frac{v_2}{v_1}\right)^{me} + 1.2\frac{\left(1+\frac{s_2 v_2^2}{s_1 v_1^2}\right)^4}{\left(1+\frac{s_2}{s_1}\right)^3}\right] - 20\log R + \Delta V + 10\log\left[\left(\frac{\rho_i}{\rho_{ISA}}\right)^2\left(\frac{c}{c_{ISA}}\right)^4\right]$ where v_1 is the jet speed at the entrance of the nozzle, v_2 the jet speed at the nozzle exit, τ_1 the inlet temperature of the nozzle, τ_2 the temperature at the nozzle exit, ρ_i the density of air, ρ_1 the atmospheric density at the entrance of the nozzle, ρ_{ISA} the atmospheric density at ground, s_1 the entrance area of the nozzle hydraulic engine, s_2 the emitting surface of the nozzle hydraulic engine, d_1 the inlet diameter of the nozzle hydraulic engine, $V_e = v_1[1-(V/v_1)\cos(\alpha_p)]^{2/3}$ the effective speed (α_p is the angle between the axis of the motor and the axis of the aircraft), R the source observer distance, w the exponent variable defined by $w = \dfrac{3(V_e/c)^{3.5}}{0.6+(V_e/c)^{3.5}} - 1$, c the sound velocity (m/s), me the exhibiting variable depending on the type of aircraft: $me = 1.1\sqrt{\dfrac{s_2}{s_1}}, \quad \dfrac{s_2}{s_1} < 29.7; me = 6.0, \quad \dfrac{s_2}{s_1} \geq 29.7$, the term $\Delta V = -15log(C_D(M_c, \theta)) - 10log(1 - Mcos\theta)$, means the Doppler convection when $C_D(M_c, \theta) = [(1 + M_c cos\theta)^2 + 0.04M_c^2]$, M the aircraft Mach Number, M_c the convection Mach Number: $M_c = 0.62(v_1 - Vcos(\alpha_p))/c$, θ is the Beam angle. The objective formula above could be written in the following simplified form $J_{G12}(\mathbf{y}(.), \mathbf{u}(.)) = \int_{t'} g(\mathbf{y}(t), \mathbf{u}(t))dt$.

1.3. Constraints

The considered constraints concern aircraft flight speeds and altitudes, flight angles and control positions, energy constraint, aircraft separation, flight velocities of aircraft relative to the earth and the aircraft mass [4,12]. On the whole, the constraints come together under the relationship $\mathbf{k}_{1i}(\mathbf{y}_i, \mathbf{u}_i) \leq 0, \mathbf{k}_{2i}(\mathbf{y}_i, \mathbf{u}_i) \geq 0$ where
$\mathbf{k}_{1i}(\mathbf{y}_i, \mathbf{u}_i) = (\alpha_i(t) - \alpha_{if}, \theta_i(t) - \theta_{if}, \psi_i(t) - \psi_{if}, \phi_i(t) - \phi_{if}, V_{a_i}(t) - V_{aif}, X_{G_i}(t) - X_{Gif}, Y_{G_i}(t) - Y_{Gif}, Z_{G_i}(t) - Z_{Gif}, p_i(t) - p_{if}, q_i(t) - q_{if}, r_i(t) - r_{if}, \delta_{l_i}(t) - \delta_{l_if}, \delta_{m_i}(t) - \delta_{mif}, \delta_{n_i}(t) - \delta_{nif}, \delta_{x_i}(t) - \delta_{xif}, m_i(t) - m_{if})$,
$\mathbf{k}_{2i}(\mathbf{y}_i, \mathbf{u}_i) = (\alpha_i(t) - \alpha_{i0}, \theta_i(t) - \theta_{i0}, \psi_i(t) - \psi_{i0}, \phi_i(t) - \phi_{i0}, V_{a_i}(t) - V_{ai0}, X_{G_i}(t) - X_{Gi0}, Y_{G_i}(t) - Y_{Gi0}, Z_{G_i}(t) - Z_{Gi0}, p_i(t) - p_{i0}, q_i(t) - q_{i0}, r_i(t) - r_{i0}, \delta_{l_i}(t) - \delta_{l_i0}, \delta_{m_i}(t) - \delta_{mi0}, \delta_{n_i}(t) - \delta_{ni0}, \delta_{x_i}(t) - \delta_{xi0}, m_i(t) - m_{i0})$.

1.4. The two-aircraft acoustic optimal control problem

The combination of the aircraft dynamic equation, the aircraft objective function and the aircraft flight constraints, the two-aircraft acoustic optimal control problem is given as follows:

$$\begin{cases} \min_{\mathbf{u}\in\mathbf{U}} J_{G12}(\mathbf{y}(.), \mathbf{u}(.)) = \int_{t10}^{t_{1f}} g_1(\mathbf{y}_1(t), \mathbf{u}_1(t), t)dt + \int_{t20}^{t_{1f}} g_{12}(\mathbf{y}_1(t), \mathbf{u}_1(t), \mathbf{y}_2(t), \mathbf{u}_2(t), t)dt \\ + \int_{t20}^{t_{2f}} g_2(\mathbf{y}_2(t), \mathbf{u}_2(t), t)dt + \phi(y(t_f)) \\ \dot{\mathbf{y}}(t) = \mathbf{f}(\mathbf{u}(t), \mathbf{y}(t)), \mathbf{u}(t) = (\mathbf{u}_1(t), \mathbf{u}_2(t)), \mathbf{y}(t) = (\mathbf{y}_1(t), \mathbf{y}_2(t)), \\ \mathbf{k}_{1i}(\mathbf{y}_i, \mathbf{u}_i) \leq 0, \mathbf{k}_{2i}(\mathbf{y}_i, \mathbf{u}_i) \geq 0, \forall t \in [t_{10}, t_{2f}], t_{10} = 0, \mathbf{y}(0) = \mathbf{y}_0, \mathbf{u}(0) = \mathbf{u}_0 \end{cases} \tag{4}$$

where g_{12} shows the aircraft coupling noise function and J_{G12} is the SEL of the two A300-aircraft.

2. The numerical processing

The problem as defined in the relation (4) is an optimal control problem [5,7] with instantaneous constraints. The fourth order Runge-Kutta method is used to solve the differential system [8]. This method is chosen because of its higher order while avoiding the disavantages of Taylor methods requiring the evaluation of partial derivatives of f.

Algorithm 1:

(1) Let us subdivide the time interval $[t_0, t_f]$ as $h = t_{n+1} - t_n = \frac{t_f - t_0}{N}$, where N is the number of samples in numerical schema.

(2)
$$\begin{array}{l} \min_{u\in\mathbf{U}} J_{12}(\mathbf{y}_n, \mathbf{u}_n) \\ \mathbf{l}_1 = h\mathbf{f}(t_n, \mathbf{y}_n, \mathbf{u}_n), \mathbf{l}_2 = h\mathbf{f}(t_n + \frac{h}{2}, \mathbf{y}_n + \frac{\mathbf{l}_1}{2}, \mathbf{u}_n), \mathbf{l}_3 = h\mathbf{f}(t_n + \frac{h}{2}, \mathbf{y}_n + \frac{\mathbf{l}_2}{2}, \mathbf{u}_n), \\ \mathbf{l}_4 = h\mathbf{f}(t_n + h, \mathbf{y}_n + \mathbf{l}_3, \mathbf{u}_n), \mathbf{y}_{n+1} = \mathbf{y}_n + \frac{1}{6}(\mathbf{l}_1 + 2\mathbf{l}_2 + 2\mathbf{l}_3 + \mathbf{l}_4), t_{n+1} = t_n + h \\ \mu_1\mathbf{k}_1(\mathbf{y}(t_n), \mathbf{u}(t_n), t_n) = 0,\ \mu_2\mathbf{k}_2(\mathbf{y}(t_n), \mathbf{u}(t_n), t_n) = 0,\ \mu_1 \leq 0,\ \mu_2 \geq 0 \\ Write\ t_{n+1} = t_n + h, \mathbf{y}_{n+1}, 0 \leq n \leq N. \end{array} \tag{5}$$

(3) Stop.

This algorithm is implemented by AMPL. The Nonlinear Interior point Trust Optimization solver "KNITRO" is called on to extract the optimal dynamic solution of the two-aircraft optimal control problem. The numerical results and the optimality convergence characteristics are presented in the following section.

3. Numerics results

Figure 1 shows the aircraft trajectories and speeds characterized by a part of constant flight level followed by a continuous descent till the aircraft touch point. Constraints on speeds are considered, allowing a subsequent landing on the same runway. The maximum altitudes considered are 3500 m and 4100 m for the first and second aircraft. The approach duration is 600 s for the first aircraft and 690 s for the second. The aircraft

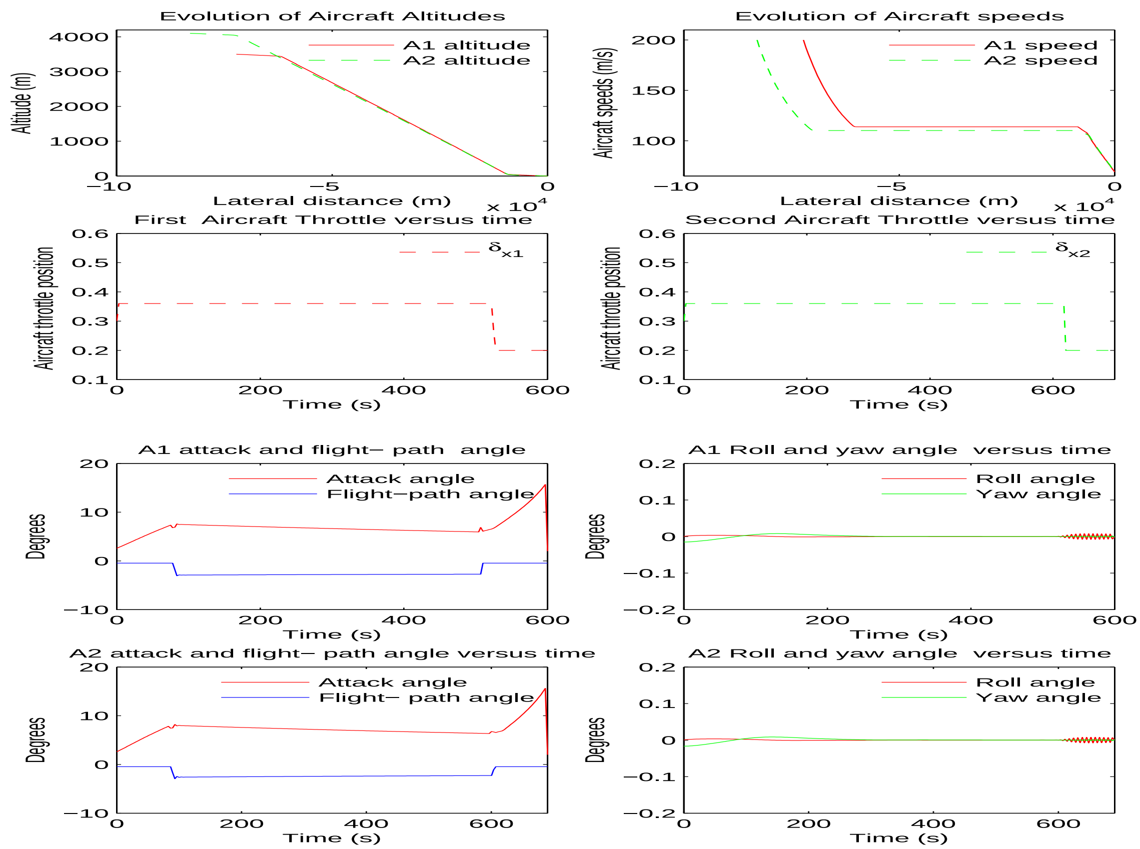

FIGURE 1. Aircraft altitudes, speeds, throttles and flight-path angles

speeds decrease from 200 m/s to 69 m/s. This shows the aircraft trajectory resulting from the two trajectories combination. Figure 1 shows also the two-aircraft flight-angles and throttles evolution versus time as recommended by ICAO during aircraft landing. As specified in this figure, the aircraft roll angles oscillate around zero. The flight-path angles are negative and bang-bang . They keep the recommended position for aircraft landing procedures. The attack angles stand between $2°$ and $20°$. Since the trajectory of the aircraft is aligned with the runway, the yaw angle are small as shown in Figure 1.

Figure 2 shows the noise levels when the optimization is applied and the solutions obtained. The observation positions are $(-20000\ m, -20000\ m, 0\ m)$ for $AONL_1$, $(-19800\ m, -19800\ m, 0\ m)$ for $AONL_2$,..., $(-200\ m, -200\ m, 0\ m)$ for $AONL_{10}$. In this figure, $AONL$ means Aircraft Optimal Noise Level. As specified, noise levels increase and are maximum when the observation point lies below the aircraft. Noise levels decrease gradually as the aircraft moves away from the observation point. This is in good agreement with [15, 16]. By comparison, this result is also close to standard values of jet noise on approach as shown by Harvey [2, 11, 14].

4. Conclusion

In this paper, a mathematical model have been developped for noise reduction in the case of two approaching aircraft landing in succession on the same runway. An algorithm for solving the optimal control problem has been developed. Theoretical considerations and practices of a direct method and the Runge-Kutta discretization scheme are used. This discretization schema is chosen because it is of sufficiently high order and it does not

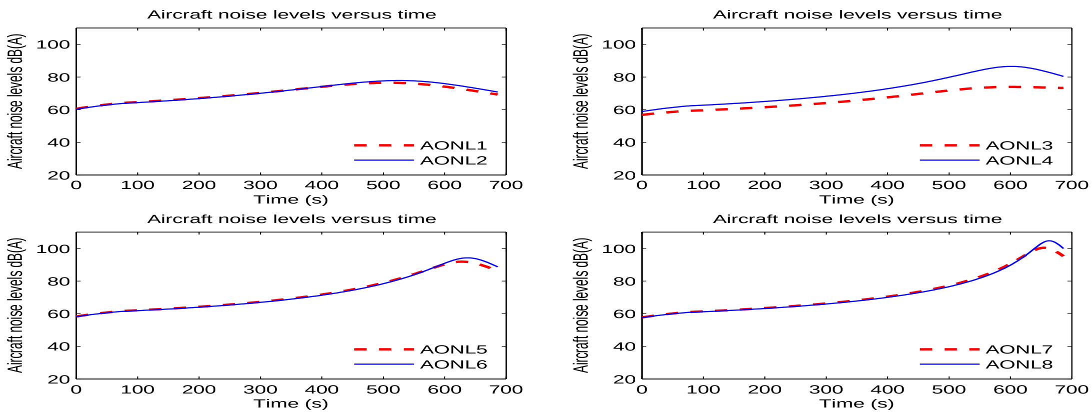

Figure 2. Aircraft optimal noise levels

require computation of the partial derivatives of f. KNITRO is applied to perform a number of numerical experiments. An optimal local solution to the discretized problem is found through a global convergence. The obtained trajectories exhibit optimal characteristics and are effective where the noise reduction is concerned.

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