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WORKSHOP ON LOW VELOCITY FLOWS: APPLICATION TO LOW MACH AND LOW FROUDE REGIMES

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Abstract. Workshop on low velocity flows: contributions

In order to describe a flow with heat transfer and at a low velocity, one usually focuses on incompressible models (zero Mach number and no thermal dilation) or on compressible models (involving acoustic waves). But each of them presents drawbacks in the simulation of such flows. On the one hand, incompressible models are not relevant anymore when heat transfers become significant. On the other hand, compressible models – which can account for heat transfers – suffer a lack of accuracy when the Mach number is very small due to the diversity of time scales (material vs. acoustic). Acoustic waves can however be neglected in many applications like in nuclear reactors in nominal or even some accidental regimes. It must be underlined that it is not the case for any application: for instance acoustic phenomena are crucial in combustion modelling.What is stated above for low Mach number flows also applies to geophysical flows when the Froude number goes to zero.

Several strategies have been developed in the literature to address theoretical and numerical issues raised by low velocity flows. A first strategy consists in deriving intermediate models (between compressible and incompressible) which are dedicated to the specific regime at stake. They are referred to as *low Mach number* models (or dilatable models). Acoustic waves are filtered out but heat transfers are still taken into account. A second strategy is aimed at determining the origin of the loss of accuracy/robustness of standard numerical schemes designed for compressible flows. They are then adapted to the specific regime (*low Mach* schemes or *all Mach* schemes). Another strategy is eventually based on coupling methods between compressible and incompressible models. The variable domains of application of each model depend on the order of magnitude of the Mach number.

The goal of the workshop was to present a range of applications where such issues occur: astrophysics, combustion, nuclear/solar energy production or geophysics. Gathering scientists involved in theory, numerics and experiments enabled to report problems and to provide a state of the art of the literature. Experts and young researchers, from academics and industry, gave 14 talks:

- Ann Almgren (Lawrence Berkeley Lab, USA): *Low Mach Number Modeling of Stratified Astrophysical Flows*;
- Emmanuel Audusse (LAGA, Université Paris 13): *Godunov type schemes for low Froude flows with Coriolis force*;
- Didier Bresch (LAMA, Université de Savoie): *On some models for bifluid flows*;
- Anne Charmeau (DEN/DANS/DM2S, CEA Saclay): *Low Mach number flow in plate-type fuel cores*;

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- Benoit DESJARDINS (FMJH): *Mathematical analysis of low Mach number flows for some single and two phase models*;
- Mathieu GIRARDIN (CMAP, École Polytechnique): *An all-regime Lagrange-Projection like scheme for 2D homogeneous models for two-phase flows on unstructured meshes*;
- Jonathan JUNG (LMAP, Université de Pau et des Pays de l'Adour): *A low Mach correction for the Godunov scheme applied to the linear wave equation with porosity*;
- Carine LUCAS (MAPMO, Université d'Orléans): *Asymptotic limits of the Shallow Water equations*;
- Sebastian NOELLE (IGPM, RWTH Aachen, Germany): *On the stability of IMEX schemes for singular hyperbolic PDE's*;
- Martin PARISOT (ANGE, Inria Paris Rocquencourt): *Low-velocity scheme for hyperbolic conservation laws with constraints*;
- Pablo RUBIOLO (IN2P3, CNRS Grenoble): *High temperature thermalhydraulics modeling of a Molten salt: application to the molten salt nuclear reactor*;
- Anouar MEKKAS and Arthur TALPAERT (DEN/DANS/DM2S, CEA Saclay and CMAP): *CDMATH library and low-Mach models applied to two-phase flows with Adaptive Mesh Refinement*;
- Adrien TOUTANT (PROMES, Université de Perpignan): *Turbulent kinetic energy transfers in low-Mach wall-bounded flows*;
- Marie-Hélène VIGNAL (Mathematics Institute of Toulouse, University Toulouse 3): *Finite volumes schemes preserving the low Mach number limit for the Euler system.*

A poster session with seven contributions was also held to complete the workshop.

The workshop took place at University Paris Descartes (lab. MAP5) in November 5th-6th 2015. It gathered more than a hundred researchers from Europe and the United States, from both academic and industrial world. It was funded thanks to the NEEDS project call for interdisciplinarity, AMIES, FMJH, Inria, RDM-IDF, UFR 929-UPMC and UPD. We address our warmest thanks to all the speakers and participants of the workshop, to all the funders and Marie-Hélène Gbaguidi of laboratory MAP5 for her considerable help for the organization of the workshop.

In this volume, six participants proposed to present their last results upon low Mach and low Froude issues, their possible link to phase transition and their potential applications to nuclear flows. These papers range from theoretical studies on the PDEs involved, describing precisely the asymptotics between several models, to the development of new efficient numerical schemes to describe these equations, as well as works on the modelling of the thermohydraulics for some applications.

The third and the fifth papers provide a theoretical asymptotic analysis for compressible Navier-Stokes equations and phase transition models in the low Mach number limit. More precisely, in the third paper, the authors introduce an original relative entropy for compressible Navier-Stokes equations with density dependent viscosities, which allows to prove some mathematical results related to the weak-strong uniqueness. Moreover, they show the convergence of the viscous shallow water equations to the inviscid shallow water equations in the vanishing viscosity limit and further prove convergence to the incompressible Euler system in the low Mach limit. The fifth paper focuses on the derivation of a system for unidimensional granular flows, which captures the transitions between compressible and incompressible phases, from compressible Navier–Stokes equations with singular viscosities and pressure. This singular limit between the two systems can then be seen as an analogue of the low Mach number limit for fluid with pressure dependent viscosity.

In the first two papers, as well as in the fourth one, numerical schemes accurate for low Mach number flows are proposed. More precisely, the first paper focuses on the inaccuracy of the classical Godunov scheme at low Froude number and introduces a way to modify it to recover a correct accuracy. In the second paper, in order to take into account the gravity in simulations of low Mach number flows, a new modification of the diffusion matrix in the context of Roe-type schemes is suggested. The fourth paper of this volume presents the analysis of an alternative numerical algorithm to solve for compressible drift-flux model, which is both accurate and robust at low Mach number on staggered grids.

In the sixth paper, physical considerations on the modeling of the thermohydraulics in a Molten Salt Fast Reactor are presented, enlightening the intrinsic multi-scale multi-physics character of the flow.